Color superconductivity and its electromagnetic manifestation

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A collection of the physical observables, related to the electromagnetic properties of a nucleon, to investigate the non–perturbative quantum fluctuations in the strong interaction vacuum state under the influence of at least one close by (in energy density) color superconducting phase found in several QCD motivated model calculations, are studied. It is shown that the spontaneous breaking of the electromagnetic gauge symmetry in the color superconducting phase of strong interaction can result in relatively clean signals in high energy processes, especially in the semi-leptonic deep inelastic scattering ones, due to a kind of electromagnetic induced strong interaction. A new type of mechanism, which is a generalization of the Higgs one, through which the local electromagnetic gauge symmetry is spontaneously broken by a spontaneous breaking of the global baryon (nucleon) number conservation, is revealed. A model independent assessment of the question of how far is the color superconducting phase of the strong interaction from its vacuum phase is made by studying currently available experimental data on the electromagnetic responses of a nucleon at high energies. It is shown that based on our current knowledge about a nucleon, it is quite likely that there is at least one color superconducting phase for the strong interaction that is close enough to the vacuum state so that its effects can even be seen in high energy processes besides heavy ion collisions.

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I. INTRODUCTION

Diquark condensation in the strong interaction ground states at asymptotically high baryon density is a very like possibility [1] since the dominant one gluon exchange interaction between quarks in such a situation is attractive in the color triplet channel which causes the BCS instability [2,3] at the Fermi surface for the quarks. At zero and low densities compared to the nuclear saturation one, the study of the phenomenon in QCD, now called *color superconductivity*, is more difficult due to the fact that one has to deal with non-perturbative and finite density effects. It is quite apparent that the strong interaction vacuum is not color superconducting at the present day condition in a large and uniform region in space since color is known to be confined. The following question can nevertheless be asked: *how far away (in energy density) is certain type of metastable color superconducting phase, called a virtual phase, of the hadronic vacuum from the stable one at the present day condition?*. It is speculated in Refs. [4–6] that certain kind of color superconducting phase exists, which has an energy density close to that of the normal phase for the strong interaction vacuum in which the chiral symmetry is spontaneously broken down. Whether of not such a speculation reflects reality is not a question that can be easily answered theoretically based on QCD since unlike at high densities, the QCD Lagrangian at low density is dominated by non-perturbative effects. Lattice study of QCD at finite density is still facing difficulties. The properties of color superconducting phases inside a nuclear matter with low or intermediate baryonic density was investigated in the literature based on four fermion interaction models [5,4,6–8], which start in the early 90s and on instanton motivated models [9–11] recently, which is currently actively studied. However model studies will not be able to determine with confidence when does certain kind of color superconductivity will appear as the density increases.

The question can be answered using experimental means at high energy due to the fact that for high energy processes, which probe the small distance properties of the hadronic system, even a close by metastable vacuum phase can contribute due to non-perturbative quantum fluctuations that sample the contributions from the quasiparticles of the metastable phase of the vacuum [7,12,13]. Direct detection of the virtual phase of the vacuum on the vacuum state itself is very difficult if possible at all. Certain finite hadronic system is needed to serve as a medium in order for significant effects of the possible color superconducting virtual phase of the strong interaction vacuum state to manifest. One of the best hadronic systems for that purpose is a single nucleon. Besides the theoretical simplicity of a nucleon, this is also mainly due to the fact that a nucleon much better known experimentally compared to a heavy nucleus so that anormalies with less uncertainties can be found. On the contrary, our combined theoretical and experimental knowledge about a heavy nucleus a lot fuzzy for our purpose.

The effects of the possible virtual color superconducting phase of the hadronic vacuum state can be observed as long as the virtual phase is close enough to the stable normal phase for hadronic systems at zero density and temperature. This is due to the fact that the concentration of matter/energy inside of a nucleon generate significant signal which

is going to be discussed in detailed in the sequel. A nucleon in either of the above mentioned situations is called a "superconducting nucleon" in the rest of the paper. It should not be confused with the naive pictures, which require that there is in fact a true color superconducting phase or there are certain kind of bound state of diquark or quark–quark clustering inside of a nucleon, which, despite of the fact that are frequently used, may be disfavored theoretically [14]. Albeit this later extreme pictures are logically covered by this paper they are not neccessary ones (see also [15]). Work in the above mentioned direction had been done in Ref. [16–18], which lead us to belief that there could be in fact a close by virtual color superconducting phase for the strong interaction vacuum. This work provides a more systematic and model independent study of the possibility.

Since color is confined in nature at the present day condition, it is extremely difficult, if possible at all, to measure the color conductivity induced phenomenon of the hadronic systems to make a direct assessment concerning whether or not a color superconducting phase is contributing or not in a given hadronic system. On the other hand, the electromagnetic (EM) superconductivity of a system can be probed using electrically charged particles like charged leptons and hadrons since the electromagnetic charge is not confined.

An earlier study were carried out [18] concerning the high energy EM properties of a nucleon if it is superconducting in the above mentioned sense. The investigation is continued into the deep inelastic scattering (DIS) processes of leptons and nucleons here by trying to explain, in an essentially model independent way, a collection of old and new anomalies about the nucleon in a logically coherent way. These anomalies are about the peculiar behavior of the elusive "Pomeron" in hadron–hadron collision and in DIS at small Bjorken scaling variable x and in certain exclusive processes. The purpose of this work is not to contradict the current explanations of some these problems in perturbative QCD. It tries to provide a complementary view, based on Regge theory for strong interaction, about the physical processes in the intersection area of soft and hard scattering kinematic region. Since this work, which contains no detailed approximate computational scheme and model information, only provides constraints due to symmetry considerations that any work which properly implements the symmetries considered is expected to arrive at basically the same results.

The small x region of the nucleon structure function, where the perturbative QCD calculation is no longer strictly valid, is an intersection region in which both the perturbative and the non-perturbative phenomena play their role. We do not understand the non-perturbative QCD well enough from a first principle point of view due to the lack of an effective and practical scheme to tackle the problem. We do have, however, a phenomenologically successful one based on Regge asymptotics, which determines the power law behavior of the hadronic scattering amplitude in the large energy limits that tells us something about the small and large x behavior of the structure function. Due to the constraint imposed by unitarity, the physical hadronic amplitudes are bounded from above by the Froissart bound which requires that the total hadron–hadron scattering cross section $\sigma_{tot} \leq \text{const} \times \log^2(s)$ with s the total energy. The Reggeon exchanged that is responsible for a behavior like $\sigma_{tot} \sim \text{const} \times \log^2(s)$ is called the soft Pomeron with an effective intercept of $\alpha_{\mathcal{P}} \approx 1.0$. A high energy virtual photon interacts with the nucleon through a coupling to the charged quarks inside the nucleon. Since quarks are the basic building block of the nucleon, it is quite natural to assume that the virtual photon and nucleon Compton scattering amplitude respect the hadronic Froissart bound too, provided that the possible final state phase space of these two reactions is of the same nature.

The recent experimental observation in lepton deep inelastic scattering (DIS) and Drell–Yan processes involving a nucleon made it possible to extract the unpolarized as well as polarized structure functions F_2 [19–21] and g_1 [22–24] respectively at small x with large momentum transfer Q^2, the flavor unsymmetry of the nucleon sea quark distribution [25] and the charge symmetry breaking strength [26,27]. The observed rapid rise of the structure functions in the small x region was unexpected[1] from a straight forward extrapolation of the perturbative QCD pictures [28,29]. It turns out that the soft pomeron can not provide the rapid rise of F_2 at small x [31] either. Instead, for a $\gamma^* N$ scattering, there is a transition region in the photon energy of order $1 GeV$ after which the so called hard pomeron with an intercept of $\alpha'_{\mathcal{P}} \approx 1.4$ seems to be needed in order the reproduce the data [32]. Such a behavior can not continue all the way down to $x = 0$ if the hard pomeron is of hadronic nature since it would lead to a violation of Froissart bound due to unitarity. Therefore although two gluon model of the hard pomeron based on the QCD evolutoin equations is capable of giving the proper rise of gluon density at small x [33], the above mentioned unitarity problem remains in such an approach. Although it can be argued that the true strong interaction asymptotics is still far from our current accessible energy scale, the physics dictated by the QCD BFKL evolution equation may not be the whole story for the small x physics for reasons not related to its apparent violation of the Froissard bound [28–30].

The observation that the center line of the flavor asymmetry can not fully account for the violation of the Gottfried

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[1]More precisely, despite the fact that it can somehow be accommodated by the perturbative QCD after the fact, it was not predicted by it.

sum rule [25] means that there seems to be less than three valence quarks inside a nucleon [34] when interpreted in a straight forward way. Where is the missing quark number? The charge symmetry breaking in the nucleon structure function obtained in Refs. [26,27] also needs to be explained. The nucleon structure function F_2 extracted from high precision neutrino DIS on a nucleon [35] disagree with the one extracted from the charged lepton DIS on the same nucleon at small x ($x < 0.1$). It is still not clear what is the origin of this difference, which could be caused by experimental systematic errors or it could be of physical in nature as anticipated in Ref. [16]. If the later possibility is true, then how can such a behavior be understood? In a recent analysis of experimental data, the vector current conservation is in question [36]. Can such a situation be incorporated into the normal picture of hadronic system without certain drastic changes to our faith to symmetries? Why does the polarized structure function seems to change so rapidly which is reported in Ref. [24] and implied in Ref. [23] at small x that no theory [37] seems can explain its origin at the present?

Admittedly, any individual piece of experimental information available at the present may not be sufficiently accurate to lead to a final conclusion, a coherent theoretical study of a collection of them is expected to provide at least a stronger motivation for us to deepen our understanding of the problem both theoretically and experimentally.

The physical processes behind the spontaneous partial breaking of the EM local $U(1)$ gauge symmetry is discussed in section II. The manifestation of the possible spontaneous partial breaking of the EM local gauge symmetry at not so high density in the high energy semi-leptonic processes of hadronic systems is studied in section III. The small q^2 region of the high energy semi-leptonic scattering is discussed in section IV. The DIS region is discussed in section V. An explanation of some of the current puzzles concerning the nucleon in the DIS processes is provided. Section VI contains a discussion and a summary.

II. PARTIAL BREAKING OF A LOCAL GAUGE SYMMETRY

The elucidation of the physical processes behind the spontaneous breaking of fundamental local gauge symmetries provides an important step for not only the construction of fundamental theories of nature, like the successful standard model of electroweak interaction, but also the further understanding of the physical properties of the superconducting phase of certain condensed matter systems. The historical development is covered in Ref. [38]. The discussion of the spontaneous breaking of the EM gauge symmetry and that of the color $SU(3)$ gauge symmetry in the color superconducting phase along the traditional lines are given, e.g., in Refs. [4,5,39,40]. However the current understanding of such a process, as it is briefly described in the following subsection, stops at the situation in which the whole charge for the local symmetry involved are spontaneously broken. With the deepening of our knowledge about the structure of matter, especially the hadronic systems, new situations emerge in which the above mentioned mechanism becomes no longer suitable. They arise because the charges for the corresponding fundamental local symmetries can be decomposed as a superposition of various components that generate certain global symmetries of the system. A familiar example is the electric charge of a fundamental particle like a quark or a lepton. It can be written generically as

$$\widehat{Q}_{em} = \widehat{Q}_B + \widehat{Q}_L + \widehat{Q}_V + \ldots \tag{1}$$

in standard model, where $\widehat{Q}_B$ is the baryon number contribution to the total electric charge, $\widehat{Q}_L$ is the lepton number contribution and $\widehat{Q}_V$ is the isospin charge contribution, etc.

It is possible that the ground or vacuum state of the system does not break the whole charge Q_{em} but only its certain component which generates global rather than local symmetry transformations, like part of the baryonic charge $\widehat{Q}_B$ of the system. In such a case [5,4,7,6], the electromagnetic gauge symmetry is said to be *spontaneously partial broken*.

But before embarking on understanding the physical processes behind the spontaneous partial breaking of a local gauge symmetry, let us briefly review the relatively well known Higgs mechanism for the spontaneous full breaking of a gauge symmetry.

A. A brief review of one charge local gauge theory and associated physical states

The gauge transformation in a system possessing the corresponding gauge symmetry can be classified into two categories 1) the gauge transformation of the first kind and 2) that of the second kind. Consider the simplest case of $U(1)$ locally symmetric gauge theories. The symmetry transformations of the first kind correspond to global ones generated by the charge operator

$$\widehat{Q} = \int d^3 x \widehat{\rho}(\mathrm{x}, t = \mathrm{const}) \tag{2}$$

of the system; the transformations of the second kind corresponding to local realization of the global ones are generated by the Gauss operator

$$\widehat{q}(x) = \widehat{\rho}(x) - \nabla \cdot \widehat{\mathrm{E}}(x), \tag{3}$$

where $\widehat{\rho}(x)$ is the charge density operator and $\widehat{E}(x)$ is the "electric" field operator. The existence of a local gauge symmetry of the theory is characterized by the existence of a superselection sector in the Hilbert space of the system, which is called the space of physical states $\mathcal{H}_{phys} = \{|\,phys\rangle\}$, within which the matrix elements of $\widehat{q}(x)$ vanish, namely

$$\langle phys\,|\widehat{q}(x)|\,phys\rangle = 0. \tag{4}$$

Due to the local gauge symmetry of the system, the superselection sector $\mathcal{H}_{phys}$ remains invariant during the time evolution of the system—-no transition from and to the other sectors of the Hilbert space $\mathcal{H}'$, which are called the subspace of unphysical states, is present.

In case of the gauge symmetry of the system is spontaneously broken down, the vacuum state is not annihilated by the charge operator $\widehat{Q}$, namely

$$\widehat{Q}|\,vac\rangle \neq 0 \tag{5}$$

leading to massless Goldstone boson excitation of the symmetry breaking according to the Goldstone theorem. These Goldstone bosons do not belong to the physical superselection sector defined by Eq. 4 of the corresponding local gauge theory so that the otherwise long range force generated by the Goldstone bosons corresponding to a spontaneous breaking of the Global symmetry of the first kind are actually absent in physical processes due to the existence of the corresponding symmetry of the second kind generated by Eq. 3. From the definition of the physical states, it can be shown that the Goldstone bosons, generated by acting upon the vacuum state by the total charge operator, belong to the subspace of unphysical states $\mathcal{H}'$. Therefore the massless Goldstone bosons associated with the spontaneous breaking of the symmetry transformations of the first kind are absent in the physical processes due to the gauge symmetry. The process of decoupling of the world-be Goldstone bosons is often called the Higgs mechanism in high energy physics.

This mechanism can be presented in a way different from the more familiar text-book discussion of the Higgs mechanism, which is based upon a Lagrangian with Higgs fields appearing at the tree-level. We are interested not only in such a possibility, but also the possibility that the "Higgs" particles are not elementary ones but composite excitations of the system that are absent at the tree-level. For that purpose, a vertex functional representation is more appropriate. As it is known, if a symmetry of the first kind is spontaneously broken, then, the corresponding Goldstone bosons appear in the vector current vertex of fermions, which is required by the Ward–Takahashi identity relating the divergence of the vector current to the self-energy of the fermions in the system [5]. In this case, the charge of the fermion is only partially concentrated on the fermionic (quasi-)particle, which forms a charge core, the rest of the charge is spreaded around it due to the existence of the massless world-be Goldstone boson. This is shown graphically in Fig. 1 and expressed in the following in terms of current operator
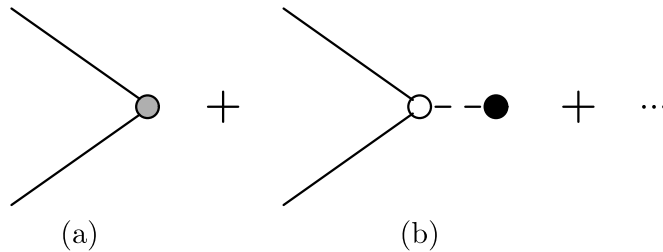

(a) (b)

FIG. 1. Various pieces for the charge of a fermionic (quasi-)particle. (a) represents the core charge contribution that concentrates on the particle. The strength of the core charge is smaller than the strength of the total charge of the original particle. The piece of charge represented by (b) is spreaded. The dashed line in (b) denotes the propagator of the Goldstone boson.

$$j^\mu(p+q,p) = j^\mu_{core}(p+q,p) + \frac{q^\mu}{q^2} j_{sprd}(p+q,p) + \ldots, \tag{6}$$

where the core current is denoted as j^μ_{core} and the spreaded piece of it, which has a strength characterized by a scalar function j_{sprd}, is longitudinal with a massless pole in q^2. The relative strength between the vector current j^μ_{core} and the scalar one j_{sprd} is determined by the current conservation, namely,

$$j_{sprd}(p+q,p) = -q_\mu j^\mu_{core}(p+q,p). \tag{7}$$

The Ward–Takahashi identity, which ensures the conservation of charge, relates the self-energy and j_{sprd} [5]. It has the form

$$j_{sprd} = \frac{e}{2}[O_3, \Sigma] = \begin{pmatrix} 0 & eD \\ -eD & 0 \end{pmatrix} \tag{8}$$

in the 8-component "real" representation for fermions with e the basic charge unit, Σ the self-energy of the fermions, O_3 the third Pauli matrix and the right hand side of the above equation acting on the upper and lower four components of the 8-components fermion spinor. The quantity D is related to the order parameter for the symmetry breaking. For a more concrete discussion, let us consider the case of scalar fermion pair condensation studied in Refs. [6,7,10,11]. Define the Goldstone boson–fermion coupling vertex as $ig\Gamma_S$ with Γ_S given by

$$j_{sprd} = e\chi\Gamma_S \tag{9}$$

and χ the order parameter (see Refs. [6,7]). Then using the graphical representation of the spreaded component of the current given in Fig. 2, one finds the value of the Goldstone boson–fermion coupling constant defined above to be

$$g = \sqrt{\frac{\chi}{A}} \tag{10}$$

with A defined in the following loop diagram [5]

$$-ieg^2\frac{q^\mu}{q^2}A\Gamma_S = \tag{11}$$

FIG. 2. The one loop graphical decomposition of the spreaded component of the charge current in terms of elementary coupling constants. The hollow dots represents the Goldstone boson and fermion coupling vertices and the grey dot is the core part of the EM charge current vertex.

Since the photon couples to the vector current of the fermions, the massless pole in the vector current vertex of the fermions in the symmetry breaking phase of the system modifies the photon behavior drastically. The fully dressed photon propagator $G_T^{\mu\nu}$ includes the self-energy insertion, namely,

$$G_T^{\mu\nu} = G_{0T}^{\mu\nu} + (G_{0T}\Pi G_{0T})^{\mu\nu} \tag{12}$$
$$\Pi^{\mu\nu} = \pi^{\mu\nu} + (\pi G_0 \pi)^{\mu\nu} + \ldots$$

$$= \quad + \quad + \ldots \tag{13}$$

in a diagrammatic representation with $\pi^{\mu\nu} = (g^{\mu\nu}q^2 - q^\mu q^\nu)\pi$ the proper self-energy of the photon and $G_0^{\mu\nu}$ the photon propagator in free space in the gauge such that $q_\mu G_0^{\mu\nu} = 0$. In the phase of the vacuum in which the EM $U(1)$ gauge symmetry is spontaneously broken, the scalar quantity π acquires a single pole in q^2, namely, $\pi = m_\gamma^2/q^2 + \ldots$ with

$$m_\gamma^2 = 4\pi\alpha_{em}g^2A^2 = 4\pi\alpha_{em}\chi A \tag{14}$$

depending on the order parameter of the symmetry breaking phase. Here α_{em} is the fine structure constant. The Dyson equation for the full photon propagator can be easily solved in the Lorentz gauge. Keeping only the pole term of π, the photon propagator has the following generic form in an infinite system [18,5]

$$G_T^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{-i}{q^2 - m_\gamma^2 + i\epsilon}. \tag{15}$$

The fermion–fermion scattering in the vector channel represented by Fig. 3 is mediated by the exchange of the media modified photon with its propagator given by Eq. 15. From Eqs. 6 and 15, it follows that only the core part of the current density j_{core}^μ participates in the scattering. Namely, the scattering amplitude T_{fi} is given by

$$-iT_{fi}^{(T)} = (ij_{core})_\mu G_T^{\mu\nu}(ij'_{core})_\nu, \tag{16}$$

which is generated by "less" charge (compared to that of the original free fermions). Therefore the scattering strength between fermions is modified by the symmetry breaking.

The spread charge current density j_{sprd} also contribute to the fermion–fermion scattering; it is mediated by an exchange of the massless Goldstone boson. Using the $U(1)$ Ward–Takahashi identity for EM [5], it can be shown that the Goldstone boson contribution to the fermion–fermion scattering is of the form

$$-iT_{fi}^{(L)} = \langle f\,|(ig\Gamma_S)|\,i\rangle\frac{i}{q^2}\langle f'\,|(ig\Gamma_S)|\,i'\rangle = (ij_{core})_\mu G_L^{\mu\nu}(ij'_{core})_\nu \tag{17}$$

with

$$G_L^{\mu\nu} = \frac{iq^\mu q^\nu}{m_\gamma^2(q^2 + i\epsilon)}, \tag{18}$$

which is represented in Fig. 3.b. Here Eqs. 9, 7, 10 and 14 are used. Therefore, the total scattering amplitude is $T_{fi} = T_{fi}^{(T)} + T_{fi}^{(L)}$, which is of the form

$$-iT_{fi} = (ij_{core})_\mu G^{\mu\nu}(ij'_{core})_\nu \tag{19}$$

with the full propagator $G^{\mu\nu}$ given by

$$G^{\mu\nu} = G_T^{\mu\nu} + G_L^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_\gamma^2}\right)\frac{-i}{q^2 - m_\gamma^2 + i\epsilon}. \tag{20}$$

The effective full propagator $G^{\mu\nu}$ for the interaction is the standard one for a massive vector particle with mass m_γ^2.

Before proceeding to the next subsection, let us make a few remarks: 1) although the core current density is used in the scattering amplitude T_{fi}, which weakens the interaction vertex, the particle exchanged has an additional longitudinal component. It is in this way that the gauge symmetry is maintained 2) the massless pole corresponding to the Goldstone boson and the massless pole of the photon disappear in the final physical scattering amplitudes. Therefore the originally non-existent Goldstone boson in the Lagrangian do not appear in the final results either; it only appear in the intermediate steps of the discussion. So it does not belong to the subspace physical states. What exchanged in the fermion–fermion scattering is a massive vector boson with standard propagator Eq. 20 for *any charged particles inside of the system* 3) it is evident from Eq. 17 that the coupling constant g between the Goldstone boson is combined with the original massless photon to generate an effective massive vector excitation. Despite that the Goldstone boson couples to the fermions with a strength g which is of order 1 due to the fact that the spontaneous partial breaking of the EM gauge symmetry is caused by strong interaction, this massive vector particle only couples to the fermions with a strength of EM interaction due to a delicate cancellation of its strong interaction effects.
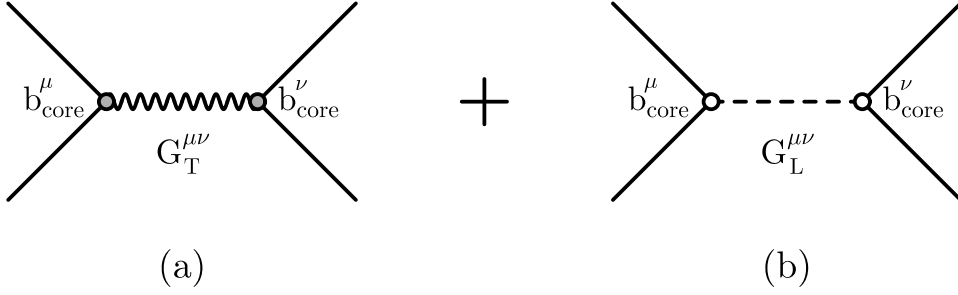
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FIG. 3. The fermion–fermion scattering through vector coupling in the phase where the gauge symmetry of EM interaction is spontaneously broken down. Figure (a) represents the scattering between particles with the same charge that generates the spontaneous broken $U(1)$ symmetry by exchange a "fully dressed photon". Figure (b) represents the scattering between particles of the same type as figure (a) by the exchange of a Goldstone boson. The effective particle that mediates the scattering of particles of the same type as figure (a) is obtained by summing figures (a) and (b). The propagator of this effective combined excitation is a massive vector one.

B. Partial breaking of local gauge symmetry in multiple charge systems

The charge operator in Eq. 2 contains only one piece since the system understudy contains only one charge [2]. The electric charge operator in the realm of elementary particle physics contains multiple pieces. In the standard model, the electric charge operator can be decomposed like in Eq. 1. In addition, the baryon number contribution $\widehat{Q}_B$ can be further decomposed into components associated with each of the three generations of hadrons, namely

$$\widehat{Q}_B = \widehat{Q}_N + \dots \qquad (21)$$

with $\widehat{Q}_N$ the contributions of the first generation of quarks consists of the light up (u) and down (d) quarks to the baryon number. The electric charge density operator can be decomposed in the same way. Although the symmetry generated by $\widehat{Q}_{em}$ has an associated local gauged one with a superselection sector of physical states, its components do not always has a local symmetry attached to them.

If there is a spontaneous breaking down of a (global) symmetry corresponding to any one component of the electric charge, then the associated Goldstone bosons do not necessarily belongs to the subspace of unphysical states (for the EM gauge symmetry) and they are expected to participate in the physical scattering processes in certain reaction channels. The Goldstone bosons still can has certain overlap with the physical superselection sector for the EM gauge symmetry in the Hilbert space of the system after the unphysical component is projected out using Eq.4.

As it is discussed above, the Goldstone boson for a single charge system is not a physical excitation. The effective excitation that mediates the interaction between the charged particles is a massive vector boson. The massless excitations decouple from the physical spectra. These occur due to a delicate cancellation of the long range interaction effects between the longitudinal components of $G_T^{\mu\nu}$ and $G_L^{\mu\nu}$ that couples to the emitter and receiver with a strength in the range of the *strong interaction* rather than that of the EM interaction. Such a cancellation is protected by the gauge invariance against higher order corrections.

For a multi-charged system like the standard model, the situation becomes a little complicated. For the scattering between two particles with the same broken charge (e.g., the nucleon charge in the light quark system), the cancellation between the long range interaction between $G_T^{\mu\nu}$ and $G_L^{\mu\nu}$ still occur [5]. Therefore a particle experiences a short ranged EM interaction when meets a particle with the same broken charge in the superconducting phase. In this case, the Higgs mechanism is functioning. What happens if two particles, A and B, with different broken charge meet? Assuming that particle A radiates a photon with a strength determined by its own electric charge and a Goldstone boson with a strength determined by its broken charge[3]. The photon will propagate with medium modified propagator

$G_T^{\mu\nu}$ on its way to particle B and the Goldstone boson will propagate with propagator $G_L^{\mu\nu}$ on its way also to particle B. Particle B receives the photon with the response determined by its electric charge and the Goldstone boson with the response determined by its own broken charge. Cancellation between the two forces induced by the longitudinal component of the medium modified EM field and that of the Goldstone boson does not happen. The residue force couples particle A and B with a strength of that of strong interaction since the Goldstone boson couples A and B in such a way. The same phenomenon happens if the role of A and B is exchanged.

This can be putted in another way. In a multi-charged system in its superconducting phase which breaks the EM gauge symmetry, the *effective* EM field strength produced by a charge particle is *observer dependent*. For a particle with the same broken charge as the source, it sees, effectively, a field of massive (short ranged in the static limit) vector particle with an strength of EM interaction. For a particle with different broken charge, what it effectively sees is two force fields: the first one is a field of massive vector particle with a intensity of EM interaction and the second one is a residue scalar field (with gradient coupling to charged particles) with a intensity comparable to that of the strong interaction that is responsible for the symmetry breaking.

Let us discuss the above statements in a more concrete fashion using the language of the previous subsection. First the Fock space of the system at the tree level is divided into subspaces, each of which contains particles with the same charge, say $\widehat{Q}_N$, that is spontaneously broken. Denoting the total EM charge current density as $J^\mu = b^\mu + \overline{J}^\mu$ with b^μ the contribution of the charge that is spontaneously broken and $\overline{J}^\mu$ the remaining charges that are unbroken. In the symmetry breaking phase, the corresponding vertex $b^\mu(p+q, p)$ for the charge current density b^μ can be decomposed in the same form as Eq. 6 so that the total vertex for the EM charge current density $J_\mu(p+q, p)$ has a form

$$J^\mu(p+q, p) = \left[b_{core}^\mu(p+q, p) + \frac{q^\mu}{q^2} b_{sprd}(p+q, p) \right] + \overline{J}^\mu(p+q, p) + \dots. \tag{22}$$

Here, the vertex for the broken charge density operator b^μ is decomposed into a sum of the core part and the spreaded part. The rest of the vertices corresponding to the unbroken charge current density is denoted by $\overline{J}^\mu(p+q, p)$. Then, using the method of the previous subsection, it is not hard to show that the fermion–fermion scattering amplitude T_{fi} in the lowest order approximation in the fine structure constant has the following general form

$$-iT_{fi} = (ib_{core})_\mu G^{\mu\nu}(ib'_{core})_\nu + (ib_{core})_\mu G_T^{\mu\nu}(i\overline{J'})_\nu + (i\overline{J})_\mu G_T^{\mu\nu}(ib'_{core})_\nu + (i\overline{J})_\mu G_T^{\mu\nu}(i\overline{J'})_\nu, \tag{23}$$

where J_μ, J'_ν denote the matrix elements of the EM current operator and $G_T^{\mu\nu}$ and $G^{\mu\nu}$ are given by Eqs. 15 and 20 respectively with $m_\gamma^2 \sim e_S^2$ where e_S is the broken charge of the charged particle. Eq. 23 indicates that only the broken charge components of the fermions are scattered by an exchange of an authentic massive vector excitation, the scattering between the rest components of the EM charge of fermions and between the broken component and the unbroken components of the EM charge operator of the fermions are mediated by exchange of massive vector excitation with its longitudinal component $G_L^{\mu\nu}$ given by Eq. 18 removed. This effective excitation contains a mixture of massive and massless excitation, which can be read out directly from Eq. 15. The massless pole in $G_T^{\mu\nu}$ will not be present explicitly on the right hand side of Eq. 23 in the vacuum case since $\overline{J}_\mu$ is a conserved current. This provides us the freedom to modify the $q^\mu q^\nu$ term of $G_T^{\mu\nu}$. It is quite natural to write Eq. 23 in an equivalent form, namely,

$$-iT_{fi} = (ib_{core})_\mu G^{\mu\nu}(ib'_{core})_\nu + (ib_{core})_\mu G^{\mu\nu}(i\overline{J'})_\nu + (i\overline{J})_\mu G^{\mu\nu}(ib'_{core})_\nu + (i\overline{J})_\mu G^{\mu\nu}(i\overline{J'})_\nu,$$
$$= (ib_{core} + i\overline{J})_\mu G^{\mu\nu}(ib'_{core} + i\overline{J})_\nu = (iJ_{core})_\mu G^{\mu\nu}(iJ_{core})_\nu, \tag{24}$$

where the propagator $G_T^{\mu\nu}$ in Eq. 23 is replaced by the full propagator $G^{\mu\nu}$.

From Eq. 24, it can be seen that in the fermion–fermion EM interaction inside the system, the core part of the EM charge participate in the interaction between charged particles exchange a massive vector boson. Since the core part of the broken charge current component of the EM charge current is not conserved, the formally similar four terms on the right hand side of Eq. 24 has different physical meaning. In the first term, the longitudinal component of $G^{\mu\nu}$ contributes since b_{core}^μ is not conserved by itself. This term can be interpreted as that both the core component and the spread component of the broken charge participate in the fermion–fermion interaction in this particular channel. The second and third terms on the right hand side of Eq. 24 have quite different physical properties. This is because $\overline{J}_\mu$ is a conserved quantity the longitudinal component of $G^{\mu\nu}$ does not actually participate in the fermion–fermion scattering in the vacuum. This is one of the reasons why it is difficult to detect the effects of the Goldstone boson in the relativistic invariant vacuum state of strong interaction.

In case of a nucleon or in a nuclear medium, the contraction of $G_L^{\mu\nu}$ with $\overline{J}_\mu$ does not give vanishing result (see the following discussion). So Eq. 23 should be used. The absence of the longitudinal contribution also means that the effects of the massless Goldstone boson is observable. Therefore we do not have a full Higgs phenomenon; the gauge symmetry is therefore considered to be "spontaneously partial broken". Let us consider the lowest order (in

α_{em}) inclusive semi-leptonic cross section. The scattering cross section σ_{in} is related to the imaginary part of the corresponding forward Compton scattering amplitude $iT^{\mu\nu}$,

$$- iT^{\mu\nu} = Z_\gamma^{-2} q^4 \int d^4x e^{iq\cdot x} \langle N | TA^\mu(x) A^\nu(0) | N \rangle, \tag{25}$$

where "T" denotes time ordering and $G^{\mu\nu}_{N;\gamma} = \langle N | TA^\mu(x) A^\nu(0) | N \rangle$ is the photon propagator in the presents of a nucleon [18]. The amputation of the external photon line *outside* of the nucleon is represented by the factor $Z_\gamma^{-2} q^4$ with Z_γ the photon wave function renormalization constant. The discussion of subsection II A for the modification of the photon propagator (see also Ref. [18]) is also applicable for $G^{\mu\nu}_{N;\gamma}$. But there are two major difference.

First, the vacuum state of the strong interaction is an infinite system in spatial extension whereas a nucleon is a finite size system. So only when the wave length of the photon is much smaller than the size of a nucleon, the photon propagator would has a similar behavior to the vacuum one [18]. Otherwise, any pole behavior in the propagators is smeared. For the simplicity of the discussion, such a smearing effects are not mentioned explicitly in most part of the discussions except in the small $Q^2 \equiv -q^2$ region since we are discussing high energy processes in which the wave length of the (virtual) photon is much smaller than the size of a nucleon.

Second, the vacuum state of the strong interaction is an isoscalar so that the proper self-energy $\pi^{\mu\nu}$ for the vacuum state,

$$- i\pi^{\mu\nu}(q) = \int d^4x e^{iq\cdot x} \langle 0 | T J^\mu(x) J^\nu(0) | 0 \rangle \Big|_{1PI}$$
$$= \int d^4x e^{iq\cdot x} \langle 0 | T \left[b^\mu(x) b^\nu(0) + \overline{J}^\mu(x) \overline{J}^\nu(0) \right] | 0 \rangle \Big|_{1PI} \tag{26}$$

contains no interference term between the current of broken charge and the current of the unbroken charge due to the isospin symmetry of the vacuum state. For the simplicity of the notation, we shall omit the "1PI" (one photon irreducible) subscript in the following. Since the proper self-energy $\pi^{\mu\nu}_N$ for a photon inside a nucleon is induced by the interaction in an isospin unsymmetric background provided by the nucleon [25], the general form of the photon proper self-energy inside a nucleon contains two additional terms corresponding to the interference terms between the current of the broken charge and that of the unbroken charge, namely,

$$- i\pi^{\mu\nu}_N(q) = \int d^4x e^{iq\cdot x} \langle N | T \left[b^\mu(x) b^\nu(0) + b^\mu(x) \overline{J}^\nu(0) + \overline{J}^\mu(x) b^\nu(0) + \overline{J}^\mu(x) \overline{J}^\nu(0) \right] | N \rangle. \tag{27}$$

The term for the broken charge of a nucleon is

$$\langle N | T b^\mu(x) b^\nu(0) | N \rangle = \langle N | \left[T b^\mu_{core}(x) b^\nu_{core}(0) + \frac{\widehat{q}^\nu}{\widehat{q}^2} T b^\mu_{core}(x) b_{sprd}(0) \right.$$
$$\left. + \frac{\widehat{q}^\mu}{\widehat{q}^2} T b(x)_{sprd} b^\nu_{core}(0) - \frac{\widehat{q}^\mu \widehat{q}^\nu}{\widehat{q}^4} T b_{sprd}(x) b_{sprd}(0) \right] | N \rangle + \dots, \tag{28}$$

where $\widehat{q}_\mu = i\partial^\mu_x$ acts only on b_{sprd} in the above equations. It can be seen that in order to maintain gauge invariance, the correlator contains not only a single massless Goldstone boson pole, but also a double massless pole that is absent in the vacuum case.

The second and third interference terms have isospin odd component that is present in a single nucleon state. Similar to Eq. 28, they can be decomposed as

$$\langle N | T b^\mu(x) \overline{J}^\nu(0) | N \rangle = \langle N | \left[T b^\mu_{core}(x) \overline{J}^\nu(0) + \frac{\widehat{q}^\mu}{\widehat{q}^2} T b_{sprd}(x) \overline{J}^\nu(0) \right] | N \rangle + \dots, \tag{29}$$

$$\langle N | T \overline{J}^\mu(x) b^\nu(0) | N \rangle = \langle N | \left[T \overline{J}^\mu(x) b^\nu_{core}(0) + \frac{\widehat{q}^\nu}{\widehat{q}^2} T \overline{J}^\mu(x) b_{sprd}(0) \right] | N \rangle + \dots. \tag{30}$$

Only a single massless Goldstone pole is present for these isospin odd terms.

There is an additional important feature compared to the vacuum case due to the fact a nucleon behaves like a medium for the photon. The matrix element of the spread current in the momentum space should be of the following form

$$b^\mu_{sprd}(p+q, p) = \frac{q^\mu + \beta P^\mu}{q^2 + \beta q \cdot P} b_{sprd}(p+q, p) \tag{31}$$

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instead of the simple form given by Eq. 6. Here P^μ is the 4-momentum of the nucleon $|N\rangle$ (the medium), p^μ is the 4-momentum of a quark. The coefficient β is determined by the nature of the symmetry breaking inside the nucleon. β is expect to decrease at large $q \cdot P$ as

$$\beta \sim \frac{m^{-1}\mu^3}{q \cdot P} \tag{32}$$

which can be estimated by a conventional finite density computation, with μ certain mass scale characterizing the concentration of a quark matter inside the nucleon and m is the mass of the nucleon. There is a corresponding change to the propagator $G_T^{\mu\nu}$ for the photon in the nucleon, which is also characterized by β. It should not be discussed quantitatively here. What is important here is the additional term βP^μ in the definition of the "longitudinal" Goldstone boson polarization, namely

$$l^\mu = q^\mu + \beta P^\mu. \tag{33}$$

The medium term βP^μ, together with the Goldstone boson, are responsible for the "extra" particle production in a high energy semi-leptonic process involving a nucleon.

In most of the expressions in the following, instead of $l^\mu/q \cdot l$, q^μ/q^2 is still used for the Goldstone boson contributions for notational simplicity. But care should be taken that the contraction of such a "q^μ" which is actually l^μ, with a conserved current should not be replaced by a zero.

C. The general picture

Two aspects are important here. The first one is related to the phenomenon of missing charge. In certain channels of reaction, like the semi-leptonic processes, only the core charge of the hadronic system couples to the external probes. Since a finite percent of the broken charge of particle is removed from its core component and added to its spreaded component in the symmetry breaking phase of the system, the system appears to these probes to contain less charge than it would normally be expected [16].

The second one is connected to the Goldstone boson degree of freedom. Let us imagine that a superconducting nucleon is hit by a high energy virtual photon γ^* or Z boson like the one shown in Fig. 4.a, then the quarks (quasiparticles) start to radiate gluons, photons and Goldstone bosons. Because $G^{\mu\nu} = G_T^{\mu\nu} + G_L^{\mu\nu}$ couples to the quarks inside the nucleon with a strength of EM interaction and $G_L^{\mu\nu}$ couples to the quarks with a strength of the strong interaction which is two order of magnitude larger that the EM one, $G_T^{\mu\nu}$ must also contain an corresponding component that couples to the quarks with a strength of the strong interaction in the superconducting phase. Therefore, the total cross section generated in Fig. 4.a is larger then that of a typical EM interaction in normal situations. For the same reason, the total cross section generated in Fig. 4.b is also larger then that of a typical EM interaction.

These two sets of graphs cancel each other if both of them are emitted by a nucleon which also contains up and down quarks. In this case, the Higgs mechanism discussed in subsection II A is realized. The nucleon–nucleon (NN) interaction is normal strong interaction plus the normal EM interaction mediated by a massive vector photon. Since the strength of the EM interaction is much smaller than the strong interaction strength, the total cross section produced by the summation of Figs. 4.a and 4.b is much smaller than that of other strong interaction processes.

In the case of charged lepton and nucleon scattering, on the other hand, the charged leptons does not couple to the Goldstone boson. So only Fig. 4.a is left. Let us write

$$G_T^{\mu\nu} = G^{\mu\nu} - G_L^{\mu\nu} \tag{34}$$

with $G^{\mu\nu}$ the massive photon propagator that couples to the quarks with a strength of EM interaction. One can ignore $G^{\mu\nu}$ in this case and write

$$G_T^{\mu\nu} \approx -G_L^{\mu\nu}. \tag{35}$$

In the vacuum, since $G_L^{\mu\nu}$ is only proportional to $q^\mu q^\nu$, $G_L^{\mu\nu}$ can not mediate force between charged leptons and quarks due to the current conservation on the lepton vertex. But inside a nucleon, $G_L^{\mu\nu}$ is proportional to $l^\mu l^\nu$. It contains terms like $\beta(P^\mu q^\nu + P^\nu q^\mu)$ and $\beta^2 P^\mu P^\nu$ due to Eq. 31. These terms do not produce a vanishing result when contracted with the lepton current. In addition, the contraction of the nucleon 4-momentum P^μ with the lepton or quark current generate a factor $q \cdot P$ at large photon energy. Therefore, due to the medium effect, the strong interaction force mediated by the Goldstone boson is independent of any mass scale even in high energy reactions. Such a strong interaction force does not, however, exist between nucleons.

Therefore, *there appears to be a new type of strong interaction inside a nuclear system when probed by systems other than another nuclear systems if a nucleon (or a nucleus) is superconducting.* As it is shown in the following, this new kind of strong interaction force may be identified with the so called "hard pomeron".
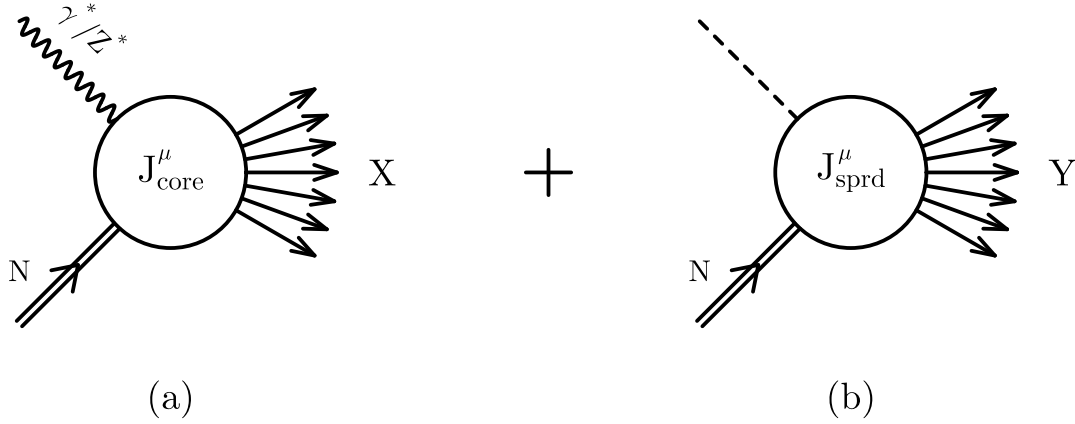

FIG. 4.   The final states generated in the neutral current semi-leptonic processes. Figure (a) represents the final states "X" generated by absorbing a (medium modified) photon with propagator given by Eq. 15. Figure (b) represents final states "Y" generated by absorbing a Goldstone boson. The strength of both of these two couplings are in fact of order of the strong interaction. If both of the photon and the Goldstone boson are emitted by another nucleon, then proper cancellation between these two sets diagrams occurs. The result is an effective massive vector photon that couples to the nucleon with a strength of EM interaction which generates far less final states compared to all other cases in which this kind of cancellation does not happen.

This specific mechanism for the explanation of the violation of the Froissart bound has several testable predictions which are explicated in the sequel.

III. SEMI-LEPTONIC PROCESSES AT HIGH ENERGIES

One way of observing the partial breaking of the EM gauge symmetry inside a hadronic system is to use the lepton–hadron scattering. Since leptons do not couple to the corresponding Goldstone boson. The general scattering amplitudes given by Eq. 23 is simplified

$$-iT_{fi} = (ib_{core} + i\overline{J})_\mu G_T^{\mu\nu}(ij)_\nu = (iJ_{core})_\mu G_T^{\mu\nu}(ij)_\nu, \tag{36}$$

where j_μ is the electro–weak current of the leptons and $J_{core}^\mu = b_{core}^\mu + \overline{J}^\mu$. So only the core part of the electro–weak current of the hadrons contributes to the semi-leptonic processes. The spreaded component of the broken charge of the nucleon is invisible. This is diagrammatically represented by Fig. 5.
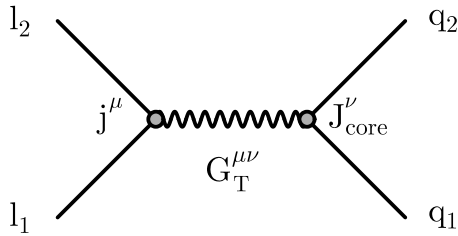
FIG. 5. The lepton–quark scattering when the EM gauge symmetry is spontaneously broken. In this case, the effective vector excitation exchanged contains not only a massive component $G^{\mu\nu}$, but also a massless component $-G_L^{\mu\nu}$. The contribution from the exchange of the Goldstone boson is absent. What the lepton sees is the core component of the broken quark baryon charge of the light quarks.

The process of the inclusive semi-leptonic DIS process is connected to the imaginary part of the forward virtual photon γ^* and Z^*- nucleon Compton scattering amplitude

$$-iT_{fi}^{\mu\nu}(p,q) = \int d^4 x e^{iq\cdot x} \langle N | TJ^\mu(x)J^\nu(0) | N \rangle, \tag{37}$$

which can be decomposed according to Eq. 28.

A. lN neutral current scattering processes

1. Gauge invariance, scattering amplitudes and structure functions

The EM interaction in the semi-leptonic lN scattering conserves parity. The kind of parity conserving decomposition of Eq. 37 is written as a sum of parity even invariant amplitudes [18]

$$T^{\mu\nu}(p,q) = \frac{1}{2m}\overline{U}(pS) \left[H_1 g^{\mu\nu} - H_2 \frac{1}{m^2} q^\mu q^\nu + H_3 \frac{1}{m^2} p^\mu p^\nu - H_4 \frac{1}{m^2}(p^\mu q^\nu + p^\nu q^\mu) + iH_5 \sigma^{\mu\nu} \right.$$
$$\left. + iH_6 \frac{1}{m^2}(p^\mu \sigma^{\nu\alpha} q_\alpha - p^\nu \sigma^{\mu\alpha} q_\alpha) + iH_7 \frac{1}{m^2}(q^\mu \sigma^{\nu\alpha} q_\alpha - q^\nu \sigma^{\mu\alpha} q_\alpha) + iH_8 \frac{1}{m^3}\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta \slashed{q} \gamma^5 \right] U(pS), \tag{38}$$

where $U(pS)$ is the spinor for a nucleon with momentum p and spin S and H_i $(i = 1, 2, \ldots, 8)$ are invariant amplitudes. If the EM gauge symmetry is spontaneously partial broken, then some of these amplitudes with at least one q^μ and/or q^ν in front carrying the Lorentz indices μ or ν develop single and double poles[4] in q^2. These parity even invariant amplitudes are H_2, H_4 and H_7. H_2 is the coefficient of $q^\mu q^\nu$, so apart from others, it constraints the contributions from the fourth term in Eq. 28 which allows it to have a double pole in q^2. H_4 and H_7 are coefficient of those terms that are linear in q^μ or q^ν, they contain contributions from the second and third terms of Eq. 28 and the second terms of Eqs. 29 and 30. Therefore, the poles of these three invariant amplitudes can be separated out

$$H_2 = \overline{H}_2 + \frac{m}{2q^2}a - \frac{m^2\nu}{q^4}b, \tag{39}$$

$$H_4 = \overline{H}_4 + \frac{m}{q^2}b, \tag{40}$$

$$H_7 = \overline{H}_7 + \frac{m\nu}{q^2}c. \tag{41}$$

The invariant amplitudes H_i $(i = 1, 2, \ldots, 8)$ are not independent of each other due to the gauge invariance expressed in terms of the Ward identity

$$q_\mu T^{\mu\nu} = 0, \tag{42}$$

which imposes constraints on them, namely,

$$m^2 H_1(q^2, \nu) - q^2 \overline{H}_2(q^2, \nu) - m\nu \overline{H}_4(q^2, \nu) = \frac{1}{2}ma(q^2, \nu), \tag{43}$$

$$m\nu H_3(q^2, \nu) - q^2 \overline{H}_4(q^2, \nu) = mb(q^2, \nu), \tag{44}$$

$$m^2 H_5(q^2, \nu) - m\nu H_6(q^2, \nu) - q^2 \overline{H}_7(q^2, \nu) = m\nu c(q^2, \nu). \tag{45}$$

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[4]They are l^μ, and $l \cdot q$ respectively for a nucleon in more precise sense. But we shall not make the distinction between them since the difference can be removed by a proper linear combination of H_i. So the generality of the discussion will not be affected.

As discussed above that in the semi-leptonic processes, only the core part of the scattering amplitude are observable. The core part of the invariant amplitude $T^{\mu\nu}$ is obtained from it by removing the poles in q^2. So

$$T_{core}^{\mu\nu}(p,q) = \frac{1}{2m}\overline{U}(pS)\left[H_1 g^{\mu\nu} - \overline{H}_2\frac{1}{m^2}q^\mu q^\nu + H_3\frac{1}{m^2}p^\mu p^\nu - \overline{H}_4\frac{1}{m^2}(p^\mu q^\nu + p^\nu q^\mu) + iH_5\sigma^{\mu\nu}\right.$$
$$\left. + iH_6\frac{1}{m^2}(p^\mu\sigma^{\nu\alpha}q_\alpha - p^\nu\sigma^{\mu\alpha}q_\alpha) + i\overline{H}_7\frac{1}{m^2}(q^\mu\sigma^{\nu\alpha}q_\alpha - q^\nu\sigma^{\mu\alpha}q_\alpha) + iH_8\frac{1}{m^3}\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta \slashed{q}\gamma^5\right]U(pS), \quad (46)$$

which, after considering the constraints Eqs. 43–45, can be reduced to

$$T_{core}^{\mu\nu}(p,q) = \mathcal{S}_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \mathcal{S}_2\frac{1}{m^2}\left(p^\mu - \frac{m\nu}{q^2}q^\mu\right)\left(p^\nu - \frac{m\nu}{q^2}q^\nu\right) + \mathcal{S}_4\frac{1}{m^2}q^\mu q^\nu + \mathcal{S}_5\frac{1}{m^2}(p^\mu q^\nu + p^\nu q^\mu)$$
$$- i\mathcal{A}_1\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}q_\alpha S_\beta - i\nu\mathcal{A}_2\frac{1}{m^2}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(S_\beta - \frac{S\cdot q}{m\nu}p_\beta\right) - i\mathcal{A}_3\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha S_\beta \quad (47)$$

with

$$\mathcal{S}_1(q^2,\nu) = -H_1(q^2,\nu) = -\frac{q^2}{m^2}\overline{H}_2(q^2,\nu) - \frac{\nu}{m}\overline{H}_4(q^2,\nu) - \frac{1}{2m}a(q^2,\nu), \quad (48)$$

$$\mathcal{S}_2(q^2,\nu) = H_3(q^2,\nu) = \frac{q^2}{m\nu}\overline{H}_4(q^2,\nu) + \frac{1}{\nu}b(q^2,\nu), \quad (49)$$

$$\mathcal{S}_4(q^2,\nu) = \frac{m}{2q^2}a(q^2,\nu) - \frac{m^2\nu}{q^4}b(q^2,\nu), \quad (50)$$

$$\mathcal{S}_5(q^2,\nu) = \frac{m}{q^2}b(q^2,\nu), \quad (51)$$

$$\mathcal{A}_1(q^2,\nu) = H_6(q^2,\nu) + \frac{\nu}{m}H_8(q^2,\nu), \quad (52)$$

$$\mathcal{A}_2(q^2,\nu) = \overline{H}_7(q^2,\nu) - H_8(q^2,\nu), \quad (53)$$

$$\mathcal{A}_3(q^2,\nu) = \frac{\nu}{m}c(q^2,\nu), \quad (54)$$

where $a(q^2,\nu)$, $b(q^2,\nu)$ and $c(q^2,\nu)$ depend on q^2 slowly.

The total virtual γ^*N cross section is related to the forward Compton scattering amplitude by the Optical theorem. For a specific polarization of the incoming (virtual) photon or Z particle with helicity λ, it is written in the form of

$$\sigma_{tot}^\lambda(\gamma^*N \to X) = \frac{4\pi^2\alpha_{em}}{mK}\varepsilon_\lambda^{\mu*}\varepsilon_\lambda^\nu W_{\mu\nu}, \quad (55)$$

where $q_\mu\varepsilon_\lambda^\mu = 0$ ($\lambda = 0,\pm$) and K as the flux factor, which is not unique when $Q^2 \neq 0$, is chosen to be $\sqrt{\nu^2 - Q^2}$. $W^{\mu\nu}$ is defined as

$$W^{\mu\nu} = \frac{1}{4\pi}\int d^4x e^{iq\cdot x}\langle pS|[J_{core}^\mu(x), J_{core}^\nu(0)]|pS\rangle. \quad (56)$$

It has the same Lorentz structure as $T_{core}^{\mu\nu}$, namely,

$$W^{\mu\nu}(p,q) = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\frac{1}{m^2}\left(p^\mu - \frac{m\nu}{q^2}q^\mu\right)\left(p^\nu - \frac{m\nu}{q^2}q^\nu\right) + W_4\frac{1}{m^2}q^\mu q^\nu + W_5\frac{1}{m^2}(p^\mu q^\nu + p^\nu q^\mu)$$
$$- iZ_1\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}q_\alpha S_\beta - i\nu Z_2\frac{1}{m^2}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(S_\beta - \frac{S\cdot q}{m\nu}p_\beta\right) - iZ_3\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha S_\beta \quad (57)$$

Optical theorem relates the imaginary part of the invariant Compton amplitudes to the structure functions in the DIS

$$W_{1,2,4,5}(q^2,\nu) = \frac{1}{2\pi}\mathrm{Im}\mathcal{S}_{1,2,4,5}(q^2,\nu + i0^+), \quad (58)$$

$$Z_{1,2,3}(q^2,\nu) = \frac{1}{2\pi}\mathrm{Im}\mathcal{A}_{1,2,3}(q^2,\nu + i0^+). \quad (59)$$

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The laboratory kinematics is defined in the usual way. The plane formed by the 3-momenta of the incident and scattering leptons is called the scattering plane. The direction defined by $\boldsymbol{q}$ is called the 3rd (or z) axis; the direction perpendicular to the the scattering plane is called the 2nd (or y) axis; the direction normal to the plane formed by the 2nd and 3rd axis is called the 1st (or x) axis. The positive quantity $Q^2 = -q^2$ is used in the following discussion.

The virtual photon 4-momentum is $q^\mu = \{\nu, 0, 0, \sqrt{\nu^2 + Q^2}\}$. The polarization vector with property $\varepsilon_\lambda^{*\mu} \varepsilon_{\lambda\mu} = 1$ and $q \cdot \varepsilon = 0$ are

$$\varepsilon_\pm^\mu = \mp \frac{1}{\sqrt{2}} \{0, 1, \pm i, 0\} \quad \varepsilon_0^\mu = \frac{1}{\sqrt{Q^2}} \{\sqrt{\nu^2 + Q^2}, 0, 0, \nu\}. \tag{60}$$

There is a fourth polarization state of the photon in the spontaneous EM gauge symmetry breaking phase. This state is some times called the longitudinal mode in the literature. The longitudinal polarization is proportional to q^μ, namely,

$$\varepsilon_s^\mu = \frac{1}{\sqrt{-Q^2}} q^\mu. \tag{61}$$

It does not contribute to the semi-leptonic processes interested in this paper.

The unpolarized total virtual $\gamma^* N$ cross sections are

$$\sigma_T = \frac{1}{2} \left(\sigma_{tot}^+ + \sigma_{tot}^- \right) = \frac{4\pi^2 \alpha_{em}}{m} \frac{1}{K} W_1, \tag{62}$$

$$\sigma_L = \sigma_{tot}^0 = \frac{4\pi^2 \alpha_{em}}{m} \frac{1}{K} \left[\left(1 + \frac{\nu^2}{Q^2} \right) W_2 - W_1 \right]. \tag{63}$$

The difference between polarized ones with the photon and nucleon spin polarization antiparallel and parallel respectively is

$$\sigma_{TT} = \frac{1}{2} \left(\sigma_{1/2} - \sigma_{3/2} \right) = \frac{4\pi^2 \alpha_{em}}{m^2} \frac{\nu}{K} \left[Z_1 - \frac{Q^2}{m\nu} Z_2 + Z_c \right], \tag{64}$$

and the interference cross section between "longitudinal" and transverse photon polarization that is perpendicular to the scattering plane is

$$\sigma_{LT} = \frac{4\pi^2 \alpha_{em}}{m^2} \frac{\sqrt{Q^2}}{K} \left[Z_1 + \frac{\nu}{m} Z_2 - \frac{\nu^2}{Q^2} Z_c \right], \tag{65}$$

where

$$Z_c = \frac{m}{\nu} Z_3. \tag{66}$$

Here

$$W_a(-Q^2, \nu) = \frac{1}{2\pi} \mathrm{Im} a(-Q^2, \nu + i0^+), \tag{67}$$

$$W_b(-Q^2, \nu) = \frac{1}{2\pi} \mathrm{Im} b(-Q^2, \nu + i0^+), \tag{68}$$

$$Z_c(-Q^2, \nu) = \frac{1}{2\pi} \mathrm{Im} c(-Q^2, \nu + i0^+). \tag{69}$$

The laboratory frame inclusive eN or μN unpolarized scattering cross section are related to these γ^* total cross section

$$\frac{d\sigma}{dE' d\Omega} = \Gamma(\sigma_T + \epsilon \sigma_L)$$

$$= \frac{\alpha_{em}^2}{8mE^2 \sin^4 \theta/2} \left[\left(2 + 2\sin^2 \theta/2 + \frac{y^2}{1-y} \right) W_2 - \frac{y^2}{1-y} W_L \right], \tag{70}$$

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where $y = P \cdot q / P \cdot k$ with k the 4-momentum of the initial lepton, for the unpolarized cross section. Here

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1}, \tag{71}$$

$$\Gamma = \frac{\alpha_{em}}{2\pi^2} \frac{E'}{E} \frac{K}{Q^2} \frac{1}{1 - \epsilon}. \tag{72}$$

If the nucleon is polarized along $\boldsymbol{q}$

$$\left(\frac{d\sigma_{-1/2}}{dE'd\Omega} - \frac{d\sigma_{+1/2}}{dE'd\Omega} \right) = 2\Gamma \sqrt{1 - \epsilon^2} \sigma_{TT}. \tag{73}$$

If the nucleon is polarized perpendicular to $\boldsymbol{q}$ and parallel to the scattering plane

$$\left(\frac{d\sigma_{-1/2}}{dE'd\Omega} - \frac{d\sigma_{+1/2}}{dE'd\Omega} \right) = 2\Gamma \sqrt{2\epsilon(1 - \epsilon)} \sigma_{LT}. \tag{74}$$

3. The two pomeron hypothesis

The presence of finite a, b and c is not a sufficient condition for the spontaneous partial breaking of the EM gauge symmetry. In fact, the same form of observable quantities in Eqs.47 and 57 with additional terms $\{\mathcal{S}_4, \mathcal{S}_5, \mathcal{A}_3\}$ and $\{W_4, W_5, Z_3\}$ can be derived from a theory in which a global $U(1)$ symmetry that is contained in the EM local gauge symmetry is explicitly broken instead of spontaneously. The difference between them lies in more details related to the existence of extra strong interaction channel in the semi-leptonic processes discussed in subsection II C.

The inclusive cross section in a semi-leptonic process are related to the imaginary part of the forward virtual photon and nucleon Compton scattering amplitude which is schematically shown in Fig. 6. Let us view this amplitude as the self-energy of the photon when propagate inside a nucleon. If the virtual photon is emitted by a nucleon, then there is a companion Goldstone boson with the proper strength that travel with the photon. Due to the Higgs mechanism, these two companion excitations effectively screen each other by canceling each others strong interaction components discussed in subsection II C so that besides other strongly interacting particles like gluons and quark–antiquark pairs, the subsequent photons and Goldstone bosons that are generated by them are also of proper portion that they cancel each other's strong interaction effects. On the other hand, if the virtual photon is emitted by a lepton, then there is no companion Goldstone boson, the passage of this virtual photon will meet greater resistance because its effective interaction with the environment inside the nucleon is in the strong interaction range. It also would frustrate the portion of the vacuum state inside the nucleon so that a disproportional number of Goldstone are generated, which in turn generate more final hadronic states after they are hadronized.

The hadronization of the Goldstone bosons (or the lack of it, see Eq. 35) are also selective. They would not hadronize into hadrons made up of up and down quarks, like the nucleons, ρ mesons, etc., since the coupling between the Goldstone boson is always screened by the virtual photon inside the system. Their coupling to other hadrons made up of quarks that do not participate in the symmetry breaking. These quarks are strange quarks, charm and bottom quarks, etc. according to the simplest scenario of color superconductivity at low density in Refs. [5,4,7,6,10,11]. Therefore the difference between the total cross section of the NN scattering and the semi-leptonic lN scattering should lie in the difference in the contributions of the strange and charm/bottom final state like the ϕ meson, J/Ψ mesons, etc., not in the regular hadronic states made of up and down quarks, like the ρ, ω, π mesons.

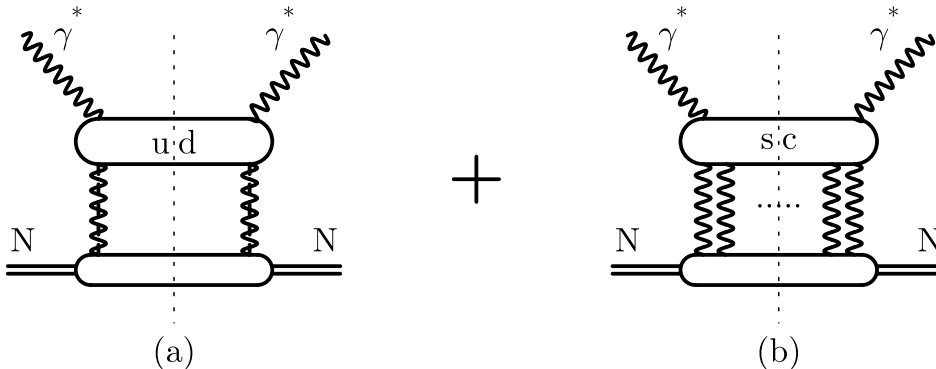(a)   +   (b)

FIG. 6. Forward Compton scattering amplitude diagrams that contribute to the multi-particle production. For the EM contribution to the NN scattering, Figures (a) and (b) are comparable in their energy dependence. In the semi-leptonic lN interaction, figure (a) contains the normal contributions and figure (b) contains extra interaction components due to the (lack of the) Goldstone boson. We shall assume the Goldstone boson can be reggonized to become the "hard pomeron".

The partial spontaneous breaking of the EM gauge symmetry inside of a nucleon is characterized by three non-vanishing pole strength of the longitudinal components of the scattering amplitude Eqs. 39–41 for *semi-leptonic processes*. Since these pole terms are absent for the EM interaction part of nucleon–nucleon scattering due to the realization of the Higgs mechanism in such a system, it would be natural to assume that the effects of the extra particles produced in a semi-leptonic lN scattering compared to the NN scattering are encoded in $a(q^2, \nu)$, $b(q^2, \nu)$ and $c(q^2, \nu)$, which should increase with the photon energy faster than that are allowed by the Froissart bound valid for the NN interaction.

According to the phenomenological success of the Regge asymptotics, the high energy behavior of $H_1, \overline{H}_2, H_3, \overline{H}_4$, H_5, H_6, $\overline{H}_7$ and H_8 follows that of the Regge asymptotics with the soft pomeron as the leading trajectory. It is known that the soft pomeron has an intercept of $\alpha_{\mathcal{P}} \sim 1$.

For the believers of two pomerons, phenomenology [31] implies that $\sigma_{\gamma^* N} \sim \nu^{\alpha_{\mathcal{P}'} - 1}$, with $\alpha_{\mathcal{P}'}$ the intercept of the "hard pomeron" of order 1.4, for sufficiently large Q^2. Let us take the two pomeron hypothesis in the present context and derive its consequences in the following.

In the Regge theory, it would be natural to assume that the high energy behavior of the leading singularity term W_b to have the following high energy behavior

$$W_b \sim \nu^{\alpha_{\mathcal{P}'} - 1}. \tag{75}$$

The high energy behavior of W_a is not known from experimental observations due to the domination of W_b in the cross section. By assuming the universality of the Regge asymptotics even in the case of hard pomeron, one can write

$$W_a \sim \nu^{\alpha_{\mathcal{P}'} - 1}. \tag{76}$$

Although there is no strong reason in doing so. W_b is an isoscalar and W_a contains isoscalar and isovector component. The high energy behavior of W_a can in principle be extracted from the data. But as a working hypothesis, I shall assume that the isovector component of W_a also satisfy Eq. 76.

The $Z_1 + Z_c$ term is related to the amplitudes for $\gamma^* \gamma^* \to N\overline{N}$ with the two transversely polarized photon having opposite helicity in the center of mass frame. The isoscalar component of each of them have the following asymptotic behavior at hight energies in the $\gamma^* N \to \gamma^* N$ channel, namely,

$$Z_{1S} \sim \nu^{\alpha_{\mathcal{P}} - 2}, \tag{77}$$
$$Z_{cS} \sim \nu^{\alpha_{\mathcal{P}'} - 2}, \tag{78}$$

where $\alpha_{\mathcal{P}}$ is the intercept of the soft pomeron and the leading trajectory for Z_c is assumed to be that of the hard pomeron also. There is no general argumentation which allows us to write down the high energy behavior for their isovector components.

In principle, the energy dependence of a, b and c and their imaginary parts may have a completely different ν dependence. For fixed Q^2, assuming the asymptotic behavior Eqs. 75, 76 and 78 hold, crossing symmetry requires that the corresponding amplitudes are of the following general form

$$a(\nu) = P_a(\nu) + \frac{2}{\pi} \int_{\nu_{th}}^{\infty} \frac{\nu' \mathrm{Im} a(\nu')}{\nu'^2 - \nu^2} d\nu', \tag{79}$$

$$b(\nu) = P_b(\nu) + \frac{2\nu}{\pi} \int_{\nu_{th}}^{\infty} \frac{\mathrm{Im} b(\nu')}{\nu'^2 - \nu^2} d\nu', \tag{80}$$

$$c(\nu) = P_c(\nu) + \frac{2}{\pi} \int_{\nu_{th}}^{\infty} \frac{\nu' \mathrm{Im} c(\nu')}{\nu'^2 - \nu^2} d\nu', \tag{81}$$

where $P_{a,b,c}(\nu)$ are polynomials of ν. The Q^2 dependence in the above equations are suppressed. $P_c(\nu) = \mathrm{const}$ and possibly non-vanishing is known (see the following discussion) from the study of the modification of GDH sum rule [18] motivated by analysis of experimental data. The form of $P_{a,b}$ remains to be determined.

The isospin structure of a, b and c and their imaginary parts can be found under the scenario of Refs. [5,6,10] and [18]. Parameter b is the strength of the double pole in q^2 in the Compton scattering amplitudes; it corresponds to the

last term of Eq. 28. Since the broken charge is an isoscalar, b is also an isoscalar. Parameter a and c are the strength of the single pole in q^2 in the Compton scattering amplitude, they contain contributions from the second and third terms in Eq. 28 and the second term in Eqs. 29 and 30. Therefore

$$a = a_S + a_V \qquad\qquad W_a = W_{aS} + W_{aV} \tag{82}$$

$$b = b_S \qquad\qquad W_b = W_{bS} \tag{83}$$

$$c = c_S + c_V \qquad\qquad W_c = W_{cS} + W_{cV} \tag{84}$$

So, the most singular piece of the Compton amplitude in q^2 can be observed in its isoscalar component and the next singular pieces of the Compton amplitude manifested themselves in the isovector component of its amplitude.

IV. THE $Q^2 \to 0$ LIMIT AND SMALL Q^2 REGION

Let us assume that all of the structure functions considered are regular in Q^2 when Q^2 is small, then following leading $1/Q^2$ behavior at small Q^2 hold

$$W_2 \to \frac{1}{\nu} W_b, \tag{85}$$

$$W_L \equiv -\frac{Q^2}{\nu^2} W_1 + \left(1 + \frac{Q^2}{\nu^2}\right) W_2 \to \frac{1}{\nu}\left[W_b + \frac{Q^2}{\nu^2}\left(W_b + \frac{\nu}{2m} W_a\right)\right], \tag{86}$$

which incorporate the possibility of spontaneous partial breaking of the EM gauge symmetry by assuming a non-vanishing $W_{a,b}$. Here all terms of order Q^2 or higher are dropped except those ones that depend on W_a and W_b since they have a quite different energy dependence from the normal ones.

If $W_{a,b} \neq 0$ and $Z_c \neq 0$, then the following limiting behavior follows

$$\sigma_L \to \frac{4\pi^2 \alpha_{em}}{m} \frac{1}{K} \frac{1}{2m}\left[W_a + \frac{2m\nu}{Q^2} W_b + \frac{2m}{\nu} W_b\right], \tag{87}$$

$$\sigma_{LT} \to \frac{4\pi^2 \alpha_{em}}{m^2} \frac{\sqrt{Q^2}}{K}\left[-\frac{\nu^2}{Q^2} Z_c\right], \tag{88}$$

when Q^2 is small.

A. Finite size effects

The longitudinal total virtual photon–nucleon scattering cross sections σ_L and σ_{LT} behave like $1/Q^2$ according to Eqs. 87 and 88. Such a behavior can not continue all the way down to $Q^2 = 0$ since a nucleon is a finite system, such a singular behavior can not be fully developed in a finite system. The rise of the cross sections as $Q^2 \to 0$ is saturated at certain small $Q^2 = Q_0^2$ value with Q_0^2 decreasing function of the photon energy ν [18]. The simplest way to represents the finite size (of the nucleon) effects is to replace $1/Q^2$ in the pole terms by the following ad hoc form

$$\frac{1}{Q^2} \to \frac{1}{Q^2 + Q_0^2}, \tag{89}$$

for the definiteness of the discussion. It turns to $1/Q_0^2$ instead of infinity in the $Q^2 \to 0$ limit. In addition, according to the discussions of Ref. [18], the maximum of the peak at the $Q^2 = 0$ position is proportional to the maximum number of the coherent interaction of the photon with the finite medium at large ν. So we can further choose a specific form for Q_0^2

$$Q_0^2 = \frac{1}{A + B\nu} \tag{90}$$

as a working hypothesis with parameters A and B slow varying function of ν. It must be emphasized that the functional forms given in Eqs. 89 and 90 are chosen only based on qualitative considerations. Other forms having the same qualitative properties may be better when detailed fitting to the experimental data are made.

Therefore, in the $Q^2 \to 0$ limit, one should replace a, b and c by

$$a \to \frac{Q^2}{Q^2 + Q_0^2} a, \tag{91}$$

$$b \to \frac{Q^4}{(Q^2 + Q_0^2)^2} b, \tag{92}$$

$$c \to \frac{Q^2}{Q^2 + Q_0^2} c, \tag{93}$$

and their imaginary part by

$$W_a \to \frac{Q^2}{Q^2 + Q_0^2} W_a, \tag{94}$$

$$W_b \to \frac{Q^4}{(Q^2 + Q_0^2)^2} W_b, \tag{95}$$

$$Z_c \to \frac{Q^2}{Q^2 + Q_0^2} Z_c. \tag{96}$$

B. The real photon limit

1. High energy behavior

After the substitutions Eqs. 91–96 are made in Eqs. 70, 73 and 74, it is easily seen that terms involving $W_{a,b}$ and Z_c are actually higher order terms in Q^2 than the leading term. The unpolarized $\gamma^* N$ cross section is

$$\sigma_{\gamma N} = \lim_{Q^2 \to 0} (\sigma_T + \epsilon \sigma_L) = \frac{4\pi^2 \alpha_{em}}{m^2} \sigma_0(0, \nu) \tag{97}$$

with

$$\sigma_0(Q^2, \nu) = -\frac{1}{2\pi} \mathrm{Im} \overline{H}_4(Q^2, \nu). \tag{98}$$

Therefore there seems to be no anomalies here. The real photon–nucleon total cross section has the same form as the one in which the EM gauge symmetry is not broken. The high energy behavior of $\overline{H}_4$ obtained from H_4 by subtracting the possible pole term in Q^2 is expected to follow the Regge asymptotics: $\overline{H}_4 \sim \nu^{\alpha_{\mathcal{P}} - 1}$ with the leading trajectory that of the soft pomeron having an intercept $\alpha_{\mathcal{P}} = 1.0808$. By assuming that the imaginary part of $\overline{H}_4$ has the same large energy asymptotics, one get

$$\sigma_{\gamma N} \sim \nu^{\alpha_{\mathcal{P}} - 1} = \nu^{0.0808}. \tag{99}$$

It agrees with the experimental observation [32] well.

The polarized γN cross section are

$$\frac{1}{2}(\sigma_{1/2} - \sigma_{3/2}) = \frac{4\pi^2 \alpha_{em}}{m^2}(Z_1 + Z_c), \tag{100}$$

$$\sigma_{LT} = 0. \tag{101}$$

2. The modification of the GDH sum rule

The first of the above equations is studied in the context of the apparent violation of the GDH sum rule [18], which states

$$\int_{\nu_{th}}^{\infty} \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{\nu} d\nu = \frac{2\pi^2 \alpha_{em}}{m^2} \kappa^2 \tag{102}$$

18

with κ the nucleon anomalous magnetic moment and ν_{th} the inelastic threshold in ν. If the EM gauge symmetry is spontaneously partial broken, then a modification of this sum rule [18] is

$$\int_{\nu_{th}}^{\infty} \frac{\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)}{\nu} d\nu = \frac{2\pi^2 \alpha_{em}}{m^2} \left(\kappa^2 + 2c_\infty\right) \tag{103}$$

in the notation of this paper. The parameter ρ_∞ defined in [18] is related to the polynomial $P_c(\nu)$ in Eq. 81 in the following way

$$\rho_\infty = \frac{1}{m^2} c_\infty = \frac{1}{m^2} \lim_{Q^2 \to 0} \lim_{\nu \to \infty} c(Q^2, \nu) \tag{104}$$

where $c_\infty = \lim_{\nu \to \infty} P_c(\nu)$. So finite modification of the GDH sum rule constrained by observation restricts the polynomial $P_c(\nu)$ to be at most a finite constant c_∞.

3. Comments

1. The current analysis of published data [41] indicates that $c_\infty \neq 0$ and mostly isovector. The most recent experimental study of GDH sum rule at MAMI in Mainz, Germany reduced the discrepancy between the theory and data to within 10% for a proton based on a preliminary analysis of the data taken. Similar study on a neutron, which is relevant to the isovector component of the difference that can even be a violation in sign, is still lacking. The theoretical extrapolation of the DIS data [42], which includes all possible photon energies also has a less than 10% difference for the GDH sum rule. Therefore whether or not the GDH sum rule is in fact violated is still an open question.

2. Since according to Eq. 84, c contains both the isoscalar and isovector components, the fact that the isoscalar component gets suppressed can also be qualitatively understood. This is because the isoscalar component relates to the second and third terms in Eq. 28, which contain b_{core}^{μ} that are reduced in strength compared to its value in the normal phase in which EM gauge symmetry is kept. On the other hand, the isovector component of c relates to the second terms in Eqs. 29 and 30, which do not contain b_{core}^{μ} and therefore is not suppressed.

3. Before continuing, a discussion regarding the subtle nature of Z_c term in Eq. 100 is necessary. Due to the separation of the pole terms from the invariant functions H_i ($i = 1, 2, \ldots, 8$) in the forward Compton scattering amplitudes, the intermediate results in deriving the modified GDH sum rule is slightly different from those in Ref. [18]. Here $\mathcal{A}_1 + c$ corresponds to mA_1 of Ref. [18]. The dominant contribution to the left hand side of the GDH sum rule is from the photo pion production through the Δ resonance. Z_c does not contain such contributions however. The reason is the pion and any other mesons made of up and down quarks belong to the hadronic subspace in which the Higgs mechanism fully operates. Since Z_c is the contribution of the Goldstone boson, it is only significant beyond the ϕ production threshold. The ϕ and J/Ψ contains no up or down quarks so that they do not couple to the Goldstone boson. Since the Goldstone boson couples to the hadronic matter with a strength of strong interaction, the lack of the contribution of the Goldstone boson means that the EM production of ϕ and J/Ψ contain an uncanceled extra component with the rate comparable to that of the strong interaction if the EM gauge symmetry is spontaneously partial broken. Therefore the effect of Z_c on the left hand side of GDH sum rule can be ignored below the ϕ production threshold, which is greater than 1 GeV, because it is of higher order in α_{em}. The contribution of these states to the sum rule are expected to be small due to the high threshold and the suppression imposed by Okubo–Zweig–Iuzuka (OZI) rule [43].

C. The small Q^2 region

The small Q^2 region is defined as the region in which the pole behaviors of a, b and c is not dominant, namely, the region in which Q^2 is not much larger than Q_0^2. This is a transition region in which the effects of the symmetry breaking terms a, b and c and all other normal terms compete with each other due to the finite size effects of a nucleon.

The more detailed description of this region is beyond the scope of this work, which is based mainly on the symmetry considerations. This is a region that can be experimentally studied in a more quantitative way.

V. THE DEEP INELASTIC, SMALL X REGION

A. Inclusive processes

In the Bjorken limit defined as the kinematic region in which $Q^2 = -q^2 \to \infty$, $\nu \to \infty$ and $x = Q^2/2m\nu =$fixed, the quark substructure of a nucleon begin to reveal itself in the structure functions of the nucleon by exhibiting scaling. This is because the large Q^2 virtual photon γ^* interacts with a quark inside a nucleon with a (light cone) time duration much smaller than the soft and coherent processes that symmetry breaking and confinement of quarks take place. Thus the virtual photon provides a snapshot of the quark distribution of a nucleon on the light cone. Combined with the asymptotic freedom, this leads to the parton model in which a factorization of the leading twist hard processes and the much slower soft processes can be factorized. In the region where Q^2 is much smaller than ν, which means small x, the light cone time duration in the current–current correlator becomes much larger than the virtual photon's wave length, the coherent processes can accumulate their strength [18]. Therefore one expects 1) the manifestation of the leading twist coherent subprocesses due to symmetry breaking, if any, in these high energy reactions and 2) non-leading twist contributions.

The scaling properties of the structure functions in the Bjorken limit make it useful to write them as

$$F_1(x, Q^2) = W_1(-Q^2, \nu), \tag{105}$$

$$F_{2,4,5}(x, Q^2) = \frac{\nu}{m} W_{2,4,5}(-Q^2, \nu), \tag{106}$$

$$F_L(x, Q^2) = \frac{\nu}{m} W_L(-Q^2, \nu) = F_2(x, Q^2) - 2xF_1(x, Q^2), \tag{107}$$

$$g_1(x, Q^2) = \frac{\nu}{m} \left[Z_1(-Q^2, \nu) + Z_c(-Q^2, \nu) \right], \tag{108}$$

$$g_2(x, Q^2) = \frac{\nu^2}{m^2} Z_2(-Q^2, \nu). \tag{109}$$

True scaling means that these structure functions are independent of Q^2 at large enough x. In QCD, they change only slowly as a function of Q^2. Such a scaling behavior of the above defined structure functions has a rather natural interpretation in terms of parton model.

In addition to these conventional structure functions, a set of new ones characterizing the symmetry breaking are defined in the following

$$F_{a,b}(x, Q^2) = W_{a,b}(-Q^2, \nu), \tag{110}$$

$$g_c(x, Q^2) = \frac{\nu}{m} Z_3(-Q^2, \nu). \tag{111}$$

Since, as it is shown above, the physical processes interested here due to the spontaneous partial breaking of the EM gauge symmetry *does not depend on a particular finite mass scale*, it is expected that scaling behavior should be present in certain forms at high energies.

It is assumed that these new structure functions change slowly with Q^2 in a logarithmic fashion in the paper for the further development of the ideas that can be experimentally examined[5].

Under such an assumption, it is expected that they are only significant in the small x kinematic region as functions of x in which coherent processes starts to build up.

The unpolarized differential cross section for the inclusive processes is

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha_{em}^2}{xQ^4} \left\{ \left[1 + (1-y)^2 \right] F_2(x, Q^2) - y^2 F_L(x, Q^2) \right\}. \tag{112}$$

The polarized differential cross section for the inclusive process proportional to the polarized total virtual photon and nucleon cross sections in Eqs. 73 and 74. σ_{TT} and σ_{LT} are related to the spin structure functions g_1 and g_2 in the following way

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[5]Note that the possible existence of a virtual phase for the strong interaction vacuum that spontaneous partial breaks the EM gauge symmetry is a *necessary* condition for making such an assumption, but it is not a sufficient one.

$$\sigma_{TT} = \frac{4\pi^2 \alpha_{em}}{mK} \left(g_1 - \frac{Q^2}{\nu^2} g_2 \right), \tag{113}$$

$$\sigma_{LT} = \frac{4\pi^2 \alpha_{em}}{mK} \frac{\sqrt{Q^2}}{\nu} \left[g_1 + g_2 - \left(\frac{m}{\nu} + \frac{1}{2x} \right) g_c \right] \tag{114}$$

with

$$g_c = \frac{\nu^2}{m^2} Z_c. \tag{115}$$

Due to the presence of the quite singular term g_c in Eq. 114, if symmetry breaking allows for $g_c \neq 0$, then the structure functions g_1 and g_2 at small x can not be extracted from the observables

$$A_1 = \frac{\sigma_{TT}}{\sigma_T}, \tag{116}$$

$$A_2 = \frac{\sigma_{LT}}{\sigma_T}. \tag{117}$$

Instead, only the following combination can be extracted

$$\tilde{g}_1 = g_1 - \frac{Q^2}{\nu^2 + Q^2} \left(\frac{m}{\nu} + \frac{1}{2x} \right) g_c = \frac{\nu^2}{\nu^2 + Q^2} F_1 \left(A_1 + \frac{\sqrt{Q^2}}{\nu} A_2 \right), \tag{118}$$

$$\tilde{g}_2 = g_2 - \frac{\nu^2}{\nu^2 + Q^2} \left(\frac{m}{\nu} + \frac{1}{2x} \right) g_c = \frac{\nu^2}{\nu^2 + Q^2} F_1 \left(-A_1 + \frac{\nu}{\sqrt{Q^2}} A_2 \right) \tag{119}$$

In the Bjorken limit, we have

$$\tilde{g}_1 = g_1 = F_1 A_1, \tag{120}$$

$$\tilde{g}_2 = g_2 - \frac{1}{2x} g_c = \frac{\nu}{\sqrt{Q^2}} F_1 A_2. \tag{121}$$

Namely, it is impossible to extract g_2 at small x from the data on A_1 and A_2 even in the true Bjorken limit.

B. Diffractive meson production

The diffractive meson production in the DIS processes provide additional means for the check various theoretical pictures for the DIS at small x. The pomeron exchange picture is proven to be a good one to describe the data [47]. The testable signature for the scenario proposed in this work are the following

1. According to the scenario proposed in this work, the diffractive production of the ρ meson, which is made up of up and down quarks, is dominated by the soft pomeron with an intercept of $\alpha_{\mathcal{P}} \sim 1.08$. This is because, as discussed above, the Hilbert space spanned by the Goldstone boson does not contribute to the ρ meson production.

2. The semi-leptonic electro- diffractive production of the ϕ and J/Ψ mesons, on the other hand, is dominated by the hard pomeron with an intercept of $\alpha_{\mathcal{P}'} \sim 1.4$ since the strange and charm quarks, like the charged leptons, have zero nucleon charge. So the cross section for the process is described by the imaginary part of Fig. 6.b, which, due to the (lack of) the contribution of the Goldstone boson in the up and down quark subspace, couples to the nucleon with the strength of strong interaction. The change of the diffractive cross section for the ϕ and J/Ψ production should follow that of the hard pomeron if there is a spontaneous partial breaking of the EM gauge symmetry according to the scenario proposed here.

3. As indicated by Eq. 7, the core part of the EM current operator for the up and down quarks is not conserved in the symmetry breaking case, it follows from Eq. 36 that current conservation shall *appear* to be violated in exclusive processes like the diffractive meson production.

C. Comments on phenomenological observations

1. If the rapid increase of measured F_2 is attributed to the symmetry breaking terms W_a and W_b, which are allowed to increase beyond the Froissart bound like Eq. 75 and 76, then we have observed behavior

$$F_2(x) \sim W_b \sim x^{1-\alpha_{\mathcal{P}'}} \approx x^{-0.4} \tag{122}$$

   at small x. It agrees with the hard Pomeron interpretation given here.

2. The structure function $F_2(x)$ is extracted from the measured unpolarized cross section corresponding to the left hand side of Eq. 112. In order to get F_2, longitudinal structure function F_L at small x, which is believed to be small, has to be known. The next-to-leading order perturbative QCD calculation of F_L is shown to incorporate the data, but it could be a self-consistent game [44] in the small x region. There could be other values for F_2 and F_L that also describe the data on the left hand side of Eq. 112. Independent experimental determination of F_L is desirable. For fixed initial lepton energy experiment, in which the quantity y is related to Q^2 by $y = Q^2/sx$ with $s = 2m\epsilon_l$ and ϵ_l the energy of the incident charged lepton, it is difficult to measure F_L since the Q^2 dependences of both F_2 and F_L are also a measured quantities.

3. If the experiment can be designed to take data with fixed Q^2 and x, then the y dependence of the cross section on the left hand side of Eq. 112, which is only quadratic in y, can be used to determine both F_2 and F_L independently. Of special interest to the scenario proposed in this work is the small x region where F_2 is shown to rise quickly. If it is indeed that the symmetry breaking terms W_a and W_b are causing the quick changes of F_2 at small x, then F_2 and F_L at small enough x will be

$$F_2 \approx \frac{1}{m} W_b, \tag{123}$$

$$F_L \approx \frac{1}{m} \left(W_b + xW_a \right). \tag{124}$$

   Let us further suppose that $W_b >> xW_a$ at small enough x, then the right hand side of Eq. 112 will be a linear rather than quadratic function of y.

4. The violation of the Gottfried sum rule is related to the unexpected behavior of the isospin odd component of F_2. It is believed to be caused by the small x region of $F_2(x)$. According to Eq. 123, the symmetry breaking effects, if present, do not contribute to it since W_b is an isospin even term (see Eq. 83). But this idealized situation was not realized in the analysis of the data because F_L is in fact unknown from direct observation. The extracted $\tilde{F}_2(x)$ under certain theoretical assumption may contain small contribution of the true F_L, namely

$$\tilde{F}_2 = F_2 + \eta F_L \tag{125}$$

   with η a small number that can has a x dependence. Since F_L contains W_a, which contains an isospin odd component (see Eq. 82), the extracted $\tilde{F}_2$ contains a term that violate the Gottfried sum rule. But this violation is expected to diminish when more information about F_L at small x is known according to the scenario proposed here. Since from Eq. 76, $F_L \sim x^{2-\alpha_{\mathcal{P}'}} \sim x^{0.6}$, the isospin odd component of $\tilde{F}_2$ contains a small piece that decreases to zero very slowly compared to the small x extrapolation used to find the discrepancy. This could be the source of the violation of the Gottfried sum rule.

TABLE I. The effective charges for the electro-weak couplings in the DIS between leptons and a nucleon/nucleus according to the standard model. $\tilde{C}_V$ and $\tilde{C}_A$ are coefficients of the vector and axial vector current operators in the hadronic weak neutral current operator. Here θ_W is the Weinberg angle. "u" and "d" denote up and down quarks respectively. The corresponding charge for an anti-quark is just opposite. The value for α in the table depends on the color and the momentum fraction x of the corresponding quark.

Quark	$\tilde{Q}(l + h \to l + h)$	$\tilde{C}_A(\nu + h \to \nu + h)$	$\tilde{C}_V(\nu + h \to \nu + h)$
u	$\alpha\frac{1}{6} + \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} - (\frac{\alpha}{3} + 1)sin^2\theta_W$
d	$\alpha\frac{1}{6} - \frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} - (\frac{\alpha}{3} - 1)sin^2\theta_W$

5. The difference between the charge lepton DIS data and the neutrino DIS data at small x ($x < 0.1$) remains after heavy target corrections are included [45]. Such a difference is anticipated [16]. The reason is because the isoscalar component of the EM charge (the nucleon number density current) of the up and down quarks contributes differently to the charge lepton neutral current DIS, neutrino neutral current and neutrino charged current DIS. For the charged lepton neutral current DIS, the isoscalar charge is $e/6$. It is $-e\sin\theta_W/3$ for neutrino neutral current DIS, which is smaller in magnitude (see Table I). Here, θ_W is the Weinberg angle. The isoscalar charge does not contribute to the neutrino charged current DIS processes.

The structure function F_2 is extracted from the experimental data based on the assumption that the core charge of quark is not modified. So The reduction of the core charge of the light quarks in the superconducting phase is expected to reduce the extracted structure function against its true value. Such a reduction is larger for the results from charge lepton neutral current DIS than the one from neutrino neutral current DIS. There is no reduction of the extracted structure function in the neutrino charged current DIS. The results for F_2 extracted from the CCFR neutrino DIS experiment and the ones obtained from various charged lepton neutral current DIS experiment do indeed show such a tendency.

6. The violation of the charge symmetry in the unpolarized structure function F_2 is due to non-linear effects. The violation is derived based on the assumption that the inclusive cross section at small x can be written as

$$F_2(x, Q^2) = \sum_i e_i^2 f_i(x, Q^2), \tag{126}$$

where the summation is over all possible flavors and $f_i(x, Q^2)$ is *independent of the charge* e_i^2.

This assumption is true in the normal cases since the fine structure constant α_{em} for EM interaction is less than 1%; higher order EM correction to F_2 can be neglected.

It is not true in case that the EM gauge symmetry is partially broken for a nucleon. Take Fig.6.b for example, the EM induced high order *effective* EM interaction between the upper and lower blocks can not be ignored. This is because the propagator $G_T^{\mu\nu}$ contains the strong interaction component $-G_L^{\mu\nu}$ (see Eq. 35). According to Ref. [5], $G_L^{\mu\nu} \sim 1/m_\gamma^2 \sim e_S^{-2}$ with e_S the broken charge[6]. But Eqs. 23 and 36 tell us that the coupling mediated by $G_T^{\mu\nu}$ is proportional to the product of the core part of the whole EM charges of the two interacting quarks inside the nucleon. So the strength of the force mediated by $G_T^{\mu\nu}$ in this case is in the strong interaction range. Higher order effects are important. The magic of the partial breaking of the EM gauge symmetry inside a nucleon is to make $f_i(x, Q^2)$ at small x to depend on the relative charge $\tilde{e}_i/e_S$ due to the unscreened contribution of the Goldstone boson, namely

$$f_i = f_i(x, Q^2; r_i), \tag{127}$$

with $r_i = \tilde{e}_i/e_S$ and $\tilde{e}_i$ the core part of the quark's EM charge.

If rigorous charge symmetry condition

$$f_{u/d}^P(x, Q^2; r) - f_{d/u}^N(x, Q^2; r) = 0 \tag{128}$$

is assumed, where the "P" denotes a proton and "N" denotes a neutron, then the compared quantities in the literature, which are

$$\Delta_1 = f_u^P(x, Q^2; r_u) - f_d^N(x, Q^2; r_d), \tag{129}$$
$$\Delta_2 = f_d^P(x, Q^2; r_d) - f_u^N(x, Q^2; r_u), \tag{130}$$

are not zero if the EM gauge symmetry is partially broken inside an nucleon. This is due to the fact that the compared quantities have different arguments.

So the finding of Refs. [26] and [27] can be interpreted as an indication of the partial breaking of the EM gauge symmetry inside a nucleon provided that Eq. 128 is true.

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[6] The isoscalar component of the up and down quark charge at low density.

7. The polarized structure functions g_1 and g_2 can not be extracted from the DIS data even in principle if the symmetry breaking term $g_c \neq 0$ since it is $\tilde{g}_1$ and $\tilde{g}_2$ (see Eqs. 118 and 119) that are related to the experimental observables A_1 and A_2. The extracted values for g_1 and g_2 also contain our uncertainty with the value of F_1 at small x even both A_1 and A_2 can be measured separately. The experimentally extracted $\tilde{g}_1$ contains a small singular piece $g_c \sim x^{-\alpha_{\mathcal{P}'}} \sim x^{-1.4}$ if the symmetry breaking effects are present. It is possible that the rapid decrease of g_1 for a neutron reported in Ref. [24] and the rapid increase of the same quantity for a proton implied in the original data of Ref. [23] are manifestations of a finite g_c for both a proton and a neutron. The most recent measurement of A_1 at SMC for x as low as 10^{-4} for a proton is plotted in Fig. 7 without the errors been displayed. The solid line that fits the data has a power law behavior of

$$A_1^P \approx 1.94 \times 10^{-3} x^{-0.23}. \tag{131}$$

Although the power -0.23 may not be taken seriously at the present stage of the experimental accuracy, one message is quite certain from that data: A_1 does not show a trend that approach to zero when $x \geq 10^{-4}$. Given $F_1 \sim F_2/x$ as a working hypothesis, it leads to $g_1^P \sim x^{-1.6}$, which is quite singular.
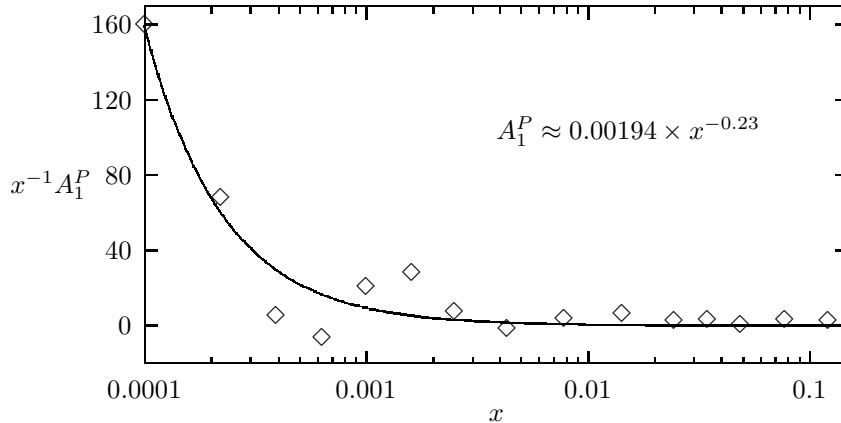

FIG. 7. The value of A_1 for a proton measured in the most recent SMC publication. It is displayed in such a way that its tendency of divergence at small x is more transparent. The solid line is a power law fit to the somewhat oscillating data points. The extracted structure function $\widetilde{g}_1^P$ is proportional to $F_2^P A_1^P / x$.

However such a singular behavior for g_1 is of high twist effects, it is expected to go a way in the $\nu \to \infty$ limit. The contribution of g_c to $\widetilde{g}_2$ is not suppressed in the Bjorken limit. In addition it is more singular than $\widetilde{g}_1$ since $\widetilde{g}_2 \sim g_c/x$.

8. The spin "crisis" is still with us. It can mean either 1) the sea quark of a nucleon is negatively polarized against the nucleon spin or/and 2) the orbital angular momentum of the quarks is finite or/and 3) the contribution of the gluons polarization through the anomalous term or/and 4) the strange content of a nucleon is finite to provide the rest of the first moment Γ_1. The recent measurement of the sea quark polarization turns out to be quite small [46]. However there is still a lack of the gluons polarization ΔG data at the present. It is possible that such a picture is correct for g_1. But the recently found rapid change of $g_1(x)$ at small x could make the crisis even more severe.

The assumption of a partial spontaneous breaking of the EM gauge symmetry could provide at least a partial understanding for the spin crisis related problems. This is because the extracted $\tilde{g}_1$ contains a singular piece $g_c \sim x^{-\alpha_{\mathcal{P}'}}$, which should give a divergent result for the first moment Γ_1 as the lower bound of the x integration is getting smaller and smaller in future measurements. The reason the present value of Γ_1 to be finite is due to the assumed normal Regge behavior extrapolation from the known data at larger x.

9. The Bjorken sum rule should be respected at least in the Bjorken limit. It is not clear whether or not the Bjorken sum rule is violated by the experimental data at finite ν and Q^2 due to the presence of g_c in the extracted $\tilde{g}_1$ in the case of symmetry breaking when more data for $\tilde{g}_1$ at smaller x is known. The reason that the Bjorken sum rule appears to be not violated in the current experimental data could mean either that it is in fact not violated or it is not violated if a normal Regge behavior for g_1 at small x is assumed. In either case, it only mean that the Bjorken sum rule is respected by the regular g_1.

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It is highly possible the the Bjorken sum rule is violated by $\tilde{g}_1$ if more experimental information about the its small x behavior is known. In that case, the isospin odd component of the imaginary part of c or Z_c is different from zero, which is allowed under the scenario proposed here (see Eq. 84). But it must be emphasized that the possible violate of the isospin odd component of the GDH sum rule and the possible violation of the Bjorken sum rule is not in one to one correspondence even in symmetry breaking case. The violation of the GDH sum rule is due to the asymptotic polynomial part of c defined in Eq. 81, which does not contribute to quantities involved in the DIS cross sections. It may or may not be zero even in the symmetry breaking case.

10. For the same reason as the ones given above, the Burkhardt–Cottingham sum rule is likely to be violated when more data about A_2 (or $\widetilde{g}_2$) is known at small x due to the singular piece proportional to g_c. It will remain to be so even in the true Bjorken limit (see Eq. 121) if the nucleon is superconducting.

11. The energy dependence of the ρ electroproduction follows that of the soft pomeron dominance in the observation. This is expected even in the symmetry breaking case. The rapid increase of the ϕ and J/Ψ production cross section [48,49] that is dominated by the hard pomeron can be incorporated in the current scenario if the partial EM gauge symmetry breaking for a nucleon is assumed. This is due to the fact that like a lepton, the strange and charm quarks contain no nucleon number. They do not couple to the Goldstone boson from EM $U(1)$ symmetry breaking. The effective EM coupling of strange and charm quark to the up and down quark sector contains a strong interaction component.

12. The current experimental data on the pp collision production of mesons seems to indicate [36] that the soft pomeron transforms as a vector that couples to the a non-conserving current. This is a natural consequences of the scenario proposed here if one assumes that there is a spontaneous EM gauge symmetry partial breaking phase for a nucleon. This is because if the soft pomeron behaves like a photon, then only the non-conserving core component of the EM charge current in the color superconducting phase contribute to the observables, as it is discussed above.

13. The scenario proposed in this work depends on the assumption that there is a nucleon long before the experimental processes take place. For those $N\bar{N}$ creation processes like in the $\gamma\gamma$ collision in the vacuum, the time for the initial $Q\overline{Q}$ creation is expected to be much faster than the collective multi-particle processes to let the excited system to cool to its ground state containing the observed $N\bar{N}$ pairs , in which the possible superconducting phase is formed. So, the proposed hard pomeron behavior should not appear [16]. This is indeed observed behavior [50].

VI. SUMMARY

The possibility to search for a close-by metastable color superconducting phase for the strong interaction vacuum state in the presence of a nucleon is studied. It is shown that the metastable color superconducting phase of the strong interaction vacuum state can manifest itself in high energy semi-leptonic neutral current electroweak interaction processes involving a nucleon through a mechanism of "spontaneous partial breaking" of the EM $U_{em}(1)$ gauge symmetry that is independent of any mass scale.

In order to achieve this, the physical processes in a system in which the EM gauge symmetry is spontaneously partial broken leading to color superconductivity is studied model independently. It is found that due the fact that the electric charge of quarks is fractional of the charge of a proton and is flavor dependent, the spontaneous partial breaking of the EM gauge symmetry in the hadronic sector caused by a diquark condensation also breaks the global baryon number conservation. The EM gauge symmetry is only spontaneously partial broken in such a case where the Goldstone bosons corresponding to the global symmetry breaking is not unphysical states in all channels of the reaction. It is also shown that for a system in which the EM gauge symmetry is spontaneously partial broken, due to the medium effects, the final phase space in a high energy semi-leptonic process contains an extra subspace compared to the allowed one due to the Froissart bound for the high energy nucleon–nucleon scattering. Such an extra subspace can be considered as spanned by the (lack of) Goldstone bosons of the the spontaneous global symmetry breaking which induces the spontaneous partial breaking of the EM gauge symmetry.

It is found here that to explain some of the puzzling behavior of the nucleon structure functions at small x, the violation of the GDH sum rule, the meson production in exclusive semi-leptonic DIS and the current non-conservation in the meson production in high energy proton–proton collisions, the assumption that a nucleon is superconducting needs to be made. The spin structure functions of the nucleons at small x is perhaps the most sensitive ones to study the spontaneous partial breaking of EM gauge symmetry inside a nucleon due to the presence of a very singular component in the experimental observables $\widetilde{g}_1(x)$ and $\widetilde{g}_2(x)$, which was seen in the most recent data at x as low as 10^{-4}

for $\tilde{g}_1(x)$. The magnitude of the effects of the spontaneous partial breaking of the EM gauge symmetry observed in the EM properties of a nucleon at high energies, if proven true, means that there is at least one color superconducting metastable phase for the hadronic vacuum state with low enough energy density[7] that can be turned into the stable one at relatively low density (compared to the nuclear saturation density) rather than at high densities. Because of the simplicity of the single nucleon system, the signal for the possible color superconducting phase of the vacuum state is relatively clear to allow a further more detailed experimental study in the existing and planed facilities measuring the high energy EM and neutral current weak responses of a nucleon at small Q^2 and at small x.

It must be emphasized that the results of this paper and earlier related ones, which concern with the virtual phases of the vacuum state, does neither uniquely imply nor disfavor the popular diquark model or quark–quark clustering model for a nucleon. These models are based on hypothetical binding/clustering between a pair of valence quarks inside a nucleon. Such a binding/clustering at valence level has no direct relation to the conclusion draw here. In fact a model for a nucleon can be build without any quark–quark clustering even in the presence of a virtual color superconducting phase for the vacuum [51].

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